Application Note:

Electronic Tilt Compensation

March 2008 **AN-00MM-004**

This application note describes the principals and application of tilt compensation to an electronic compass.

1. Principle of the compass Tilt Compensation

Electronic compass uses geomagnetism to find the azimuth or the heading direction. Let X axis of compass point forward and Y axis point to the right (Fig.1). When X and Y axes are in the horizontal plane using formula (1) we can calculated the azimuth or the heading direction angle (α).

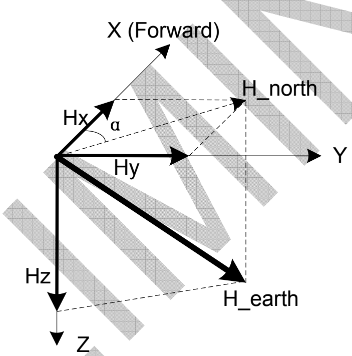

Fig.1 Geomagnetism decomposer

Azimuth=arctan(y/x) (1)

Azimuth=90 (x=0, y<0)
Azimuth=270 (x=0, y>0)
Azimuth=180-[arctan(y/x)] * 180/∏ (x<0)
Azimuth=-[arctan(y/x)] * 180/∏ (x>0, y<0)
Azimuth=360-[arctan(y/x)] * 180/∏ (x>0, y>0)

Most often compasses are not confined to a flat and level plane. This makes it more difficult to determine the azimuth, or heading direction, since the compass is not always parallel to the earth's surface. Errors introduced by tilt angles can be quite large depending on the amount of the Dip angle. A typical method for correcting the compass tilt is to use an inclinometer, or tilt sensor, to determine the roll and pitch angles. The terms roll and pitch are commonly used in aviation: roll refers to the rotation about the X, or forward direction axis, and pitch refers to the rotation about the y, or left-right, direction axis. The pitch and roll of the compass is shown in Fig.2.

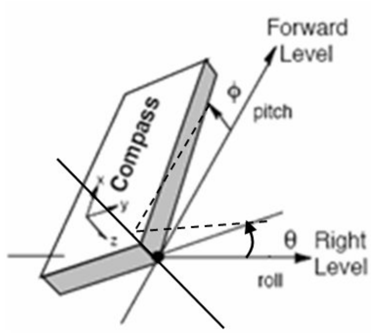

Fig.2 Pitch and Roll of the Compass

If CMx represents the compensated x direction magnetic field output and CMy is the compensated y direction magnetic field output, the tilt compensation formula for magnetic output should be (Regarding to x, y and z directions in Fig.1):

$$\begin{aligned} CMx &= Mx \cdot \cos(pitch) + My \cdot \sin(roll) \cdot \sin(pitch) + Mz \cdot \cos(roll) \cdot \sin(pitch) \\ CMy &= My \cdot \cos(roll) - Mz \cdot \sin(roll) \end{aligned} \quad (2)$$

When the accelerometer is tilted, the output of the accelerometer is related to the angle (Θ) formed by the gravity (G) vector and the accelerometer axis by a sine function:

$$Accout = G \cdot \sin\theta \quad (3)$$

So formula (2) can be simplified to formula (4):

$$\begin{aligned} CMx &= Mx \cdot \sqrt{1-Accx^2} + My \cdot Accy \cdot Accx + Mz \cdot \sqrt{1-Accy^2} \cdot Accx \\ CMy &= My \cdot \sqrt{1-Accy^2} - Mz \cdot Accy \end{aligned} \quad (4)$$

And now the azimuth is calculated by formula (5).

$$Azimuth = \arctan(CMy/CMx) \quad (5)$$

2. 3-axis Tilt compensation

Normally a 2-axis accelerometer can be used to successfully measure pitch and roll within the linear range of the accelerometer measurement domain. When compass is tilted beyond 45 degrees or less than -45 degrees the accelerometer's output will have difficulty providing accurate output due to non-linearity. Fig.3 shows the sine function's linear region.

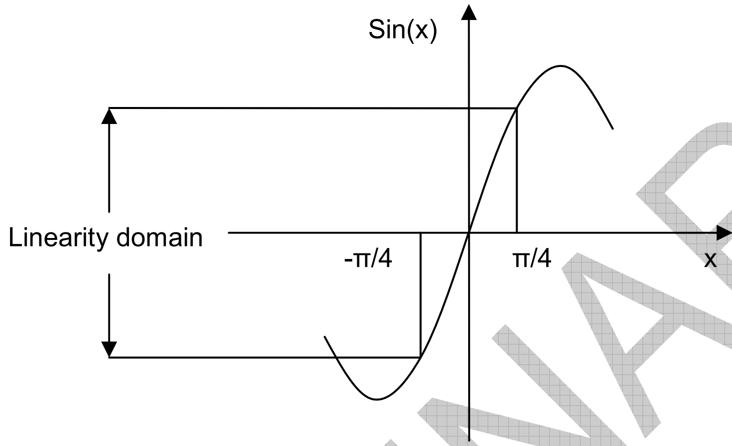

Fig.3 Linearity of the Sine function

When accelerometer inclination angles are outside the linearity domain (larger than 45 degrees or less than -45 degrees in Fig.3.), the sensitivity is reduced due to the inherent nature of a sinusoidal curve, causing a smaller accelerometer output change. When the inclination angles are greater than 60 degrees the output of the accelerometer will be even smaller. It's this sine function characteristic that leads to the inaccuracy of the accelerometer outputs in large angle domains. Hence, in large angle domains a tri-axis accelerometer tilt compensation solution is needed.

The following is the principle of 3-axis tilt compensation. We start with the relationship of 3 axes of the accelerometer in Fig.4.

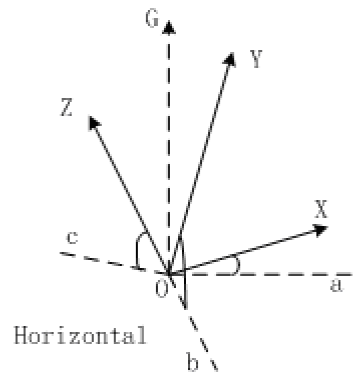

Fig.4 Relationship of 3 Accelerometer Axes

In typical real world applications, only one of the three angles formed by X, Y and Z axes with respect to the horizontal plane is larger than 45 degrees at any given time. In Fig.4 x-axis is used to measure pitch ($\angle$aOX) and y-axis is used to measure roll ($\angle$bOY). When pitch or roll angle is larger than 45 degrees z-axis can be used to recalculate this angle by the relationship which can be described by formula (6).

$$\sin^2(\angle aOX) + \sin^2(\angle bOY) + \sin^2(\angle cOZ) == 1 \qquad (6)$$

It's easy to prove this formula. If there is a vertical vector G with respect to horizontal plane, the three componential angles to x, y and z axes have the relationship:

$$\cos^2(\angle GOX) + \cos^2(\angle GOY) + \cos^2(\angle GOZ) == 1 \qquad (7)$$

Also $\angle$aOX+$\angle$GOX==$\angle$aOY+$\angle$GOY==$\angle$aOZ+$\angle$GOZ==π/2, so formula (6) can be generated from formula (7) easily.

Formula (6) can also be written as:

$$Accx^2 + Accy^2 + Accz^2 = 1 \qquad (8)$$

Knowing the relationship between the output component for each axis and the way to compensate nonlinearity error of them we can use Z axis output now to compensate the error using the expression (9). For the y-axis the process is the same.

```
if (Accx²>(Accy² + Accz²))
   if (Accx>0)
     Accx=sqrt(1-Accy²-Accz²)                    (9)
   else
     Accx=-sqrt(1-Accy²-Accz²)
```

In expression (9) Accx is the sine result of pitch angle and Accy is the sine result of roll angle. All that is needed is to judge if the angle is larger than 45 degrees or smaller than -45 degrees first and then recalculate this angle.

Put the result from formula (9) into formula (4) and the accurate CMx and CMy will be calculated and the accurate azimuth can be found in large angle domains.

Revision History

Revision	Date	Comment
1.1	19-Mar-08	Initial Draft release (Updated format)

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