Optimal Loading of Audio Transformers for Crystal Set Use

Crystal receivers have attracted for decades the attention of radio enthusiasts, mainly because of its low parts count and capability for long-distance reception when connected to an efficient antenna-ground system.

Overall crystal set design (and construction) criteria try to get the most of the RF power intercepted by the antenna into the headphones, ultimately as audio-frequency power, this power being converted into sound by the hearing element.

In recent years, utmost importance has been given to the utilization of matching audio transformers and suitable hearing devices for ultimate volume improvements in these receivers. Fig.1 shows the schematic diagram of a basic crystal set featuring an audio impedance-matching stage.

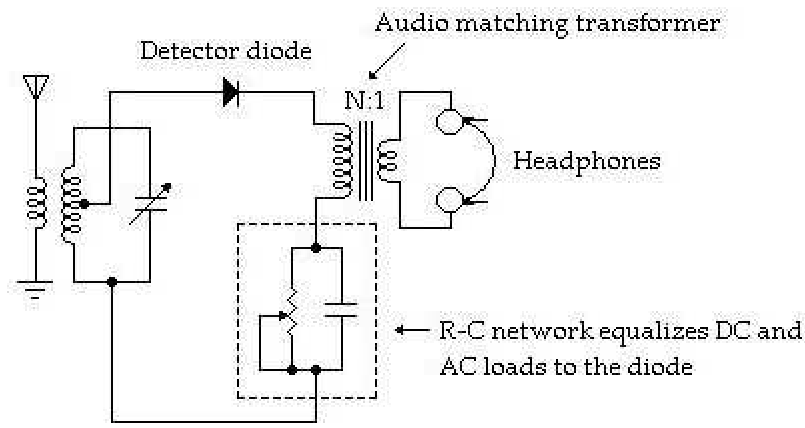


Fig.1 Basic xtal set with headphones impedance-matched to the detector

This article will present technical material the author believes could be helpful to the hobbyist when selecting a suitable audio transformer for his set or when studying utilization of the one just found in the spare parts box.

We shall begin making some basic power calculations. First, consider a sine-wave generator delivering power to a resistive load R_L, as shown in Fig.2. V_g is the peak amplitude of the source voltage and R_g is the source resistance.

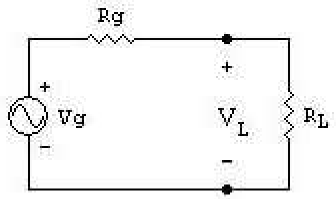


Fig.2 Generator delivering power to a resistive load R_L

The average power dissipated by R_L is:

$$P_L = \frac{V_L^2}{2R_L} \qquad \ldots(1)$$

where V_L is the peak value of the voltage across the load. V_L is computed as:

$$V_L = V_g \frac{R_L}{R_L + R_g}$$

Substitution into eq.(1) yields:

$$P_L = V_g^2 \frac{R_L}{2(R_L + R_g)^2}$$

Maximum power is delivered to the load when $R_L = R_g$. In this particular case:

$$P_L = P_{L\,MAX} = \frac{V_g^2}{8R_g} \qquad \ldots(2)$$

This is the maximum available power.

Next, consider the situation where R_L differs from R_g and we still want the maximum available power delivered to R_L. At audio frequencies, it is common practice to connect a matching transformer between the source and the load. This device permits transformation of impedance levels, such that the generator "sees" an equivalent load $R_L' = R_g$. Maximum power is then available and it will be transferred to R_L. Please refer to Fig.3.

The transformer should be a low-loss type, in order to restrict power losses to a minimum. Usually, a specially treated steel-laminated core is used when higher permeability values and very tight magnetic coupling between windings is required.

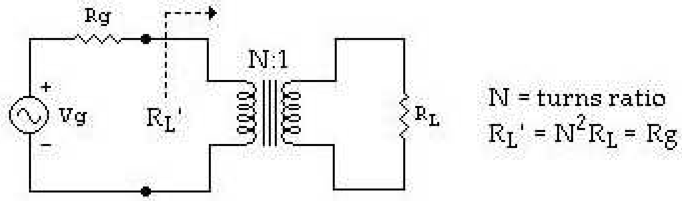


Fig.3 Audio transformer coupling the load R_L to the voltage source

In a crystal receiver, R_g represents the detector diode's output resistance at audio frequencies and R_L, the effective average impedance of a pair of 2k ohms DC resistance magnetic headphones, sound powered headphones or piezoelectric ceramic or crystal earpiece.

R_L must be matched to R_g for maximum power transfer to the hearing device.

Modeling a transformer

An equivalent network for an audio transformer can be seen in Fig.4. Here, circuit parameters have been defined in terms of the inductances L_P and L_S of the primary and secondary windings, the coupling coefficient k between these windings, stray capacitances and losses.

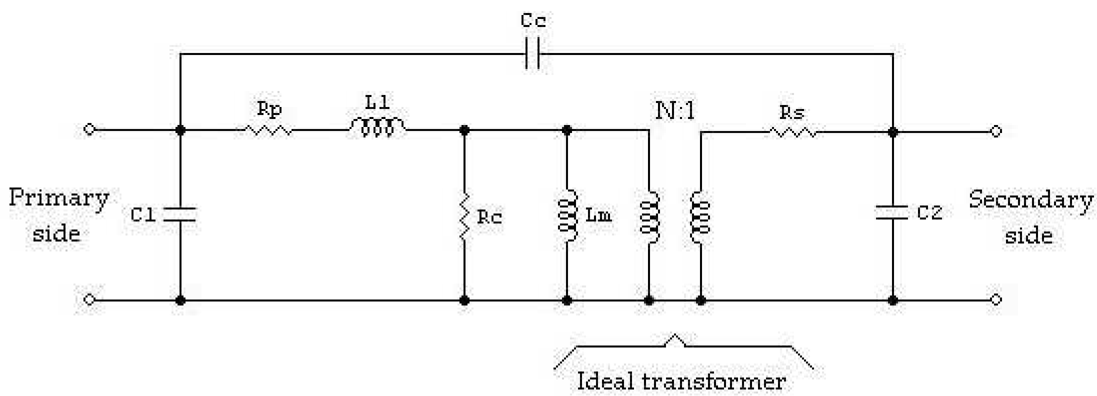


Fig.4 Equivalent circuit for an audio transformer showing components that tailor the frequency response

The following relationships apply:

$$L_m = k^2 L_p \qquad \ldots(3.1)$$

$$L_1 = \left(1 - k^2\right) L_p \qquad \ldots(3.2)$$

$$N = k\sqrt{\frac{L_p}{L_s}} \qquad \ldots(3.3)$$

- L_m is the magnetizing (or shunt) inductance. Its finite value is responsible for power losses at low audio frequencies.
- L_l is the equivalent leakage inductance referred to the primary side. It results from magnetic flux not mutually linked by the windings and contributes to losses at high frequencies.
- N is the "turns ratio".
- The copper loss (resistance) of the primary and secondary windings is represented by Rp and Rs, respectively.
- Rc represents core losses. Contributions to these losses come from eddy-currents and hysteresis behaviour.
- C_1 and C_2 are the intra-winding capacitances and Cc is the inter-winding capacitance. These three are stray (parasitic) capacitances. They also contribute to power losses at the high-frequency end of the response.
- If the windings are DC-isolated from each other, the short connecting the lower ends of the ideal transformer in the model should be substituted by a second capacitor Cc connected between the lower input and output terminals.

Selecting a suitable transformer

For crystal set use, a good transformer should have a flat response from 300Hz to 3000Hz (or better) when loaded following manufacturer´s specs. Accordingly, the following relationships should be satisfied:

$$R_g \rangle\rangle R_p$$

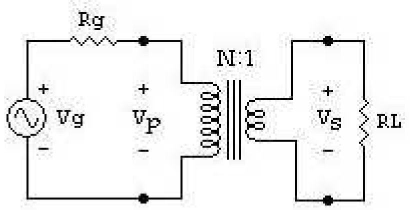
$$R_L \rangle\rangle R_s$$

$$R_c \rangle\rangle R_L' = R_g \qquad \ldots(4)$$

$$k \approx 1$$

Also, at rated loads, the effects of C_1, C_2 and Cc should be noticeable only at higher frequencies, beyond the midband.

A typical amplitude versus frequency response curve for an audio transformer is shown in Fig.5. In this figure, 0dB refers to the output level at midband frequencies. At frequencies f_L and f_H the output is down by 3dB.



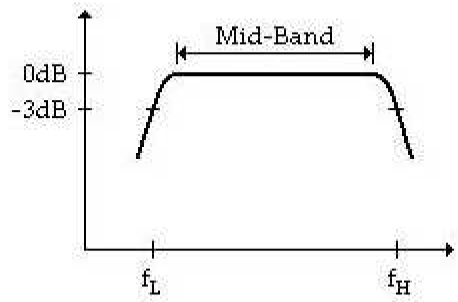
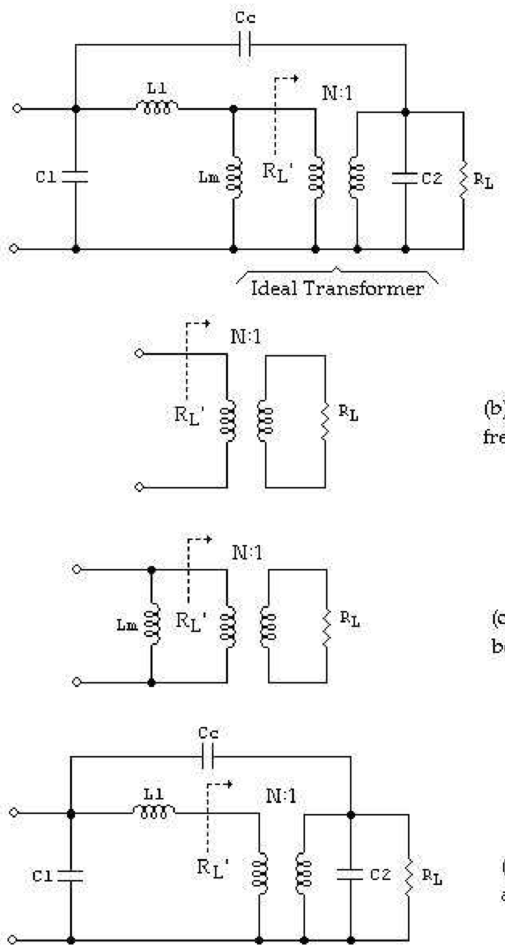


Fig.5 Typical amplitude response
of an audio transformer

For a load resistance R_L equal to or greater than the rated value $R_{L\ nominal}$, but much smaller than Rc/N^2, the transformer may be represented by the simplified models of Fig.6.



(a) Model useful when $R_{L\ nominal} \leq R_L < Rc/N^2$

(b) Model valid for midband frequencies

(c) Model valid for frequencies below the midband

(d) Model valid for frequencies above the midband

Fig.6 Simplified equivalent circuits
for the audio transformer

We are interested in knowing if a specific transformer will efficiently match the given impedance levels R_g and R_L. We would also like to know if the 300Hz~3000Hz bandwidth (minimum) will be accomplished by using this audio transformer.

If the device has been manufactured for audio frequency operation (there are reports saying that small 60Hz low-voltage power-line transformers have been successfully used as matching devices), then chances are that it will reproduce frequencies up to at least 3000Hz. However, attenuation at lower frequencies will be strongly dependent on the magnetizing inductance L_m and the value of the source resistance R_g (high-quality units may reproduce frequencies down to 50Hz +/-1dB, referenced to the midband).

The equivalent circuit shown in Fig.6.c is very valuable for evaluating transformer operation at frequencies below the midband. We will use this model for calculation of f_L, the lower -3dB frequency (please see Fig.5). At this frequency, the power delivered to the load R_L will be one half of that available in the midband, this is, it will be 3dB down.

<u>Computing the half-power point</u>

If we lower the operating frequency, eventually the primary's magnetizing inductance L_m will start shunting the available signal, reducing the output power. Recalling that at mid frequencies $V_p = V_g/2$, the half-power point of the response will be described by that frequency at which the amplitude of the primary's voltage falls to:

$$V_p = \frac{V_g}{2\sqrt{2}} \qquad \ldots(5)$$

With Fig.6.c in mind, we may draw the circuit of Fig.7 to help us in the calculation of the lower -3dB frequency.

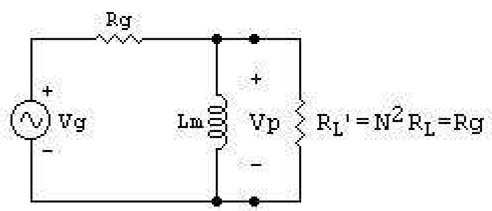


Fig.7 Equivalent circuit for calculation of the -3dB frequency

Analysis of the circuit yields for the primary's voltage:

$$V_p = \frac{V_g}{2} \cdot \frac{j\omega L_m}{\frac{R_g}{2} + j\omega L_m}$$

where ω = 2πf is the radian frequency and $j = \sqrt{-1}$.

Then, for the half-power point [eq.(5)]:

$$\frac{V_g}{2\sqrt{2}} = \frac{V_g}{2} \cdot \frac{\omega L_m}{\sqrt{\frac{R_g^{\ 2}}{4} + \omega^2 L_m^{\ 2}}}$$

or:

$$\frac{1}{2} = \frac{\omega^2 L_m^{\ 2}}{\frac{R_g^{\ 2}}{4} + \omega^2 L_m^{\ 2}}$$

Simplifying:

$$\frac{R_g}{2} = \omega L_m \qquad \ldots(6)$$

The above equation tells us that, under matched conditions, at the lower –3dB frequency the reactance of the magnetizing inductance equals one half of the source resistance.

Then:

$$\omega_{3dB} = \frac{R_g}{2L_m}$$

and:

$$f_{3dB} = \frac{1}{2\pi} \cdot \frac{R_g}{2L_m} \qquad \ldots(7)$$

If the shunt inductance L_m and the source resistance R_g are known quantities, f_{3dB} can be readily obtained. On the other hand, if L_m and the turns ratio N are known, selecting f_{3dB} will yield the corresponding values for R_g and the optimum load R_L, recalling that $R_g = N^2 R_L$.

A useful approximation for N is:

$$N = \sqrt{\frac{L_p}{L_s}} \qquad \ldots(8)$$

which requires that L_P and L_S be known. Also, being $k \approx 1$, we may write $L_m \approx L_P$.

Working out some examples

Case # 1

Some months back the author received a small audio transformer having the code ST-11 stamped on its side. No technical info was available at that time. The only known fact was that the external connection to the windings was a set of flexible green, red, white and black wires.

With the help of an audio generator and an oscilloscope some basic measurements were made. First, the green-red wires were identified as corresponding to the high-impedance winding (primary) and the white-black pair as that pertaining to the low-impedance side (secondary). Then, an approximate value for the turns ratio N was obtained. A 0.1V peak-amplitude 1kHz signal was applied to the primary, giving 0.022V peak across the unloaded secondary. This yielded a 4.54:1 voltage transformation ratio or N. However, it is recommended the turns ratio be measured under rated-load conditions (impractical at this point of our work, as we knew nothing about the impedances of this little transformer).

The author could also get a hand on a B&K 875A LCR meter for inductance and resistance measurements. The high-impedance winding measured L_P = 19.5H and DC resistive losses of Rp = 1.236k ohms. The low-impedance side showed L_S = 0.833H and DC resistive losses of Rs = 153 ohms. Applying eq.(8) a value of 4.84 for N was obtained. This value is believed to be a better approximation for N.

As mentioned before, the minimum acceptable bandwidth should be 300Hz to 3000Hz, flat. In practice, amplitude response variations of +/-1dB relative to the midband are acceptable. For the response at 300Hz to be within this tolerance, we must select 150Hz as the –3dB frequency (at two times the corner frequency, the response is within 1dB of the value found at mid frequencies).

From eq.(7) we already know that at f_{3dB} the primary's reactance equals Rg/2. At two times f_{3dB}, the reactance will equal R_g. This is a useful result, stating that at the lower end of the flat passband the primary's reactance will be equal to R_g.

We can use the above results in the following way. From the formula for a coil's reactance:

$$X_L = 2\pi fL \qquad \ldots(9)$$

the impedance of the primary winding at 300Hz (neglecting resistive losses) is found to be Xp = 36.757k ohms. The secondary winding yields a value Xs = 1.57k ohms. Accordingly, R_g should be 36.757k ohms for a –3dB frequency of 150Hz, the optimum load being R_L = 1.57k ohms.

A hearing device having an effective average audio impedance around 1.5k ohms will be matched to a detector diode's output audio impedance of 30…..40k ohms. This is likely a value for a typical germanium 1N34 diode in an average performance crystal set using a tapped detector coil. If higher load impedances are used, the –3dB frequency will be shifted upwards and there will be losses at bass frequencies.

For core losses to be neglected, the transformed load impedance should satisfy the following relationship:

$$R_L^{'} = N^2 R_L \approx \frac{L_p}{L_s} R_L = \frac{X_p}{X_s} R_L \langle\langle R_c$$

In the present case, Rc was found to be 447k ohms (the method for taking this measurement will be discussed in a future article). As a gross approximation, a value for Rc equal to 20 times $N^2R_{L\ nominal}$ may be assumed for a good audio transformer. Accordingly, the optimum values for R_g and R_L should be:

$$R_g = \frac{447}{20} = 22.35kohms$$

$$R_L = 22.35\frac{X_S}{X_P} = 0.954kohms$$

Technical data was found recently on this transformer describing it as a Philmore 20k:1k 50mw input transformer (no more info available).

Case #2
Our second real-world example deals with the Calrad 45-700 audio transformer, specified by the manufacturer as a 100k:1k matching device. Measurements were taken to verify technical data found on the Internet. Results are tabulated below.

Winding	Wire Color Code	Inductance	Copper Losses
Primary	Green-Red	Lp = 52.5H	Rp = 2.11k ohms
Secondary	Green-White	Ls = 0.55H	Rs = 69.6 ohms

Calculated reactances at 300Hz are Xp = 98.96k ohms and Xs = 1.036k ohms for the primary and secondary, respectively, very close to the rated transformed and load impedances (100k ohms and 1k ohms).

The above values suggest that, under rated loading and matched conditions, the Calrad 45-700 will yield a lower corner frequency approximately equal to 150Hz.

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