Tuning PID Controllers

Cheng-Liang Chen
PSE
LABORATORY
Department of Chemical Engineering
National TAIWAN University

Outline

- ➢ Selection of Controller Modes
- ➢ Controller Tuning Tips
- ➢ Integrated Tuning and Diagnosis
- ➢ Tuning for Quarter Decay Ratio
- ➢ Tuning for Minimum-Error Integrals
- ➢ Controller Synthesis

Tuning Map for Gain and Reset Effects

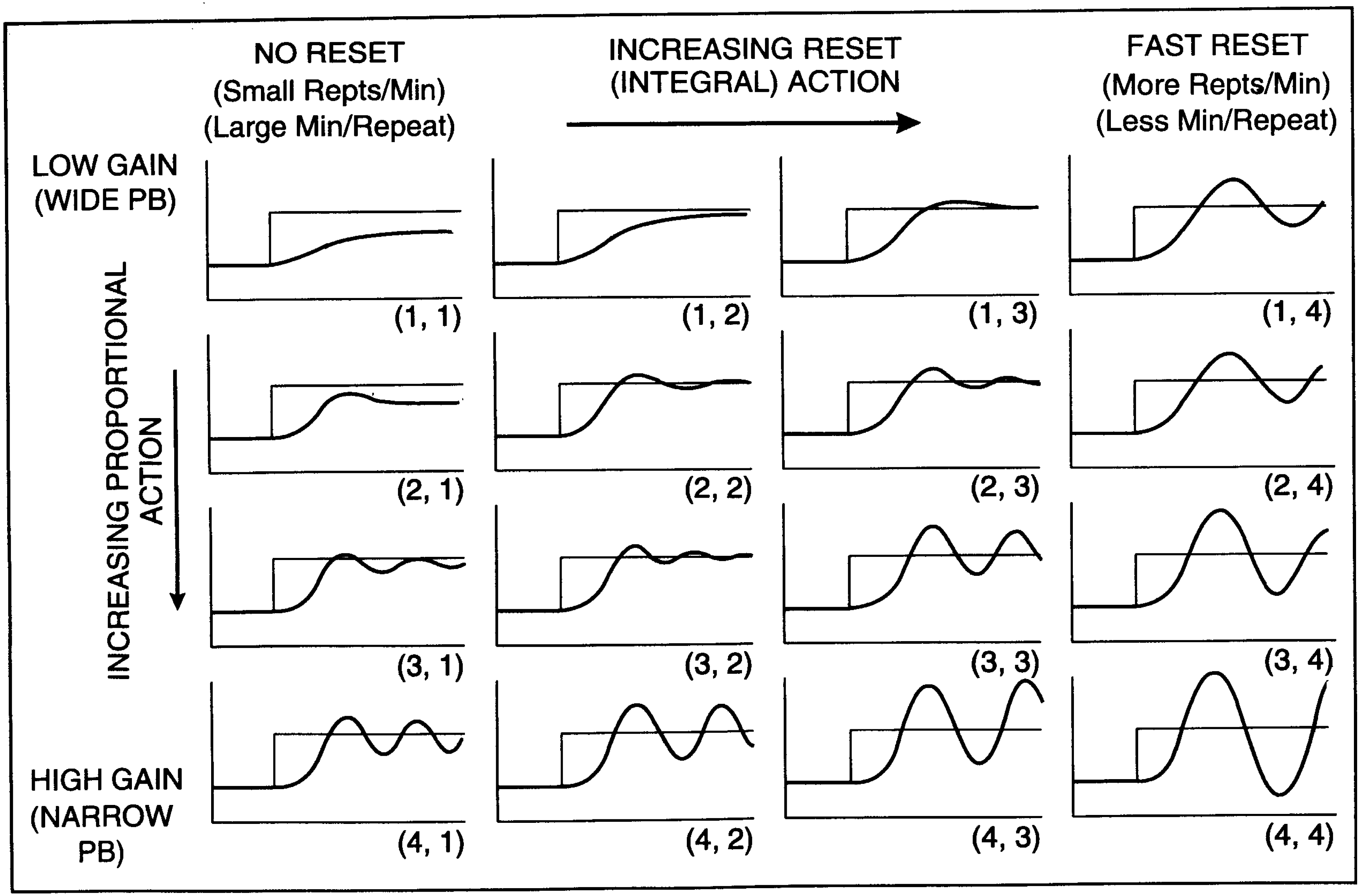


Figure 8-2: Tuning Map for Gain and Reset Effects.

The Effects of Adding Derivative

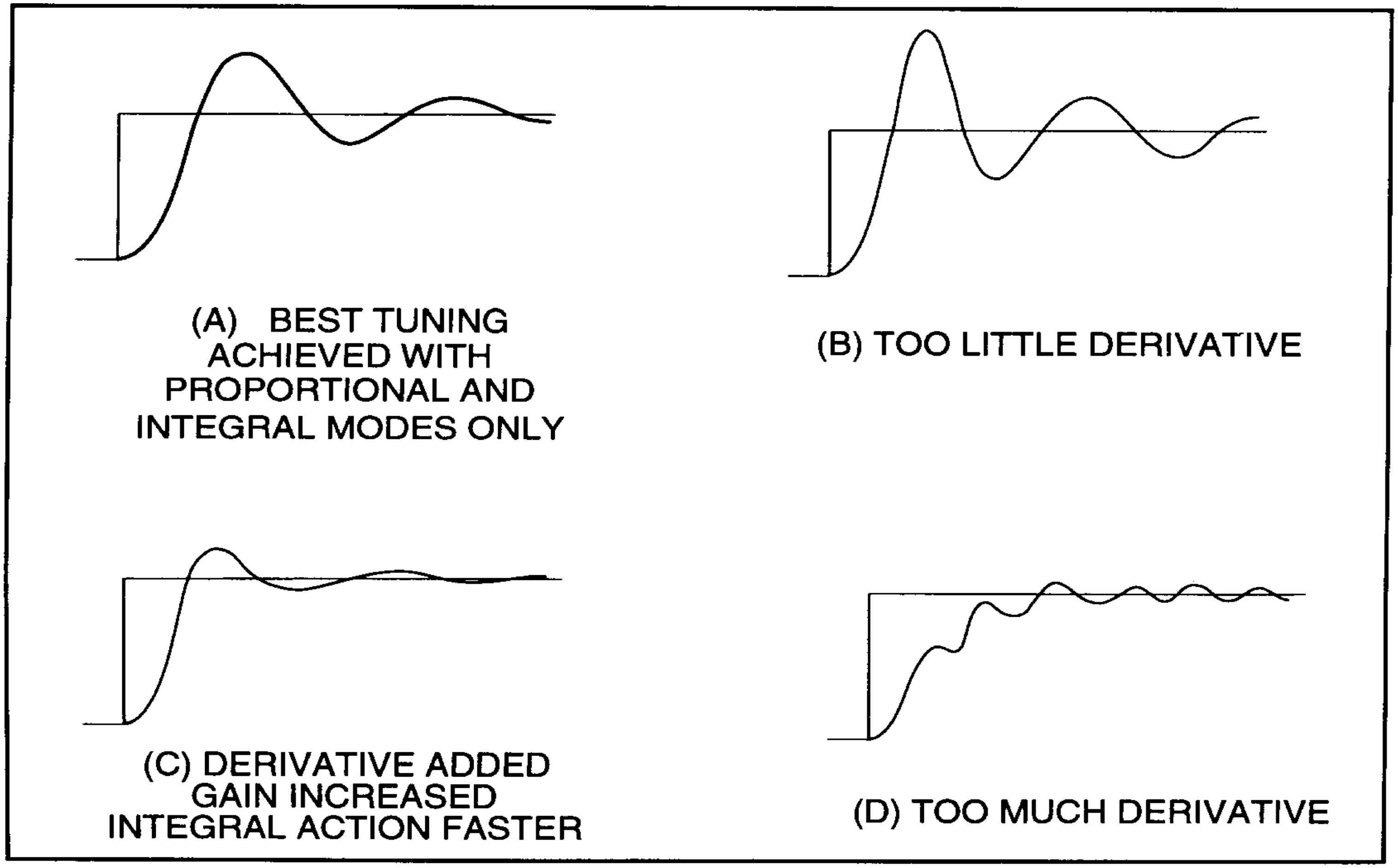


Figure 8-3: The Effect of Adding Derivative.

Selection of Controller Modes

Selection of Controller Modes
Deciding on Control Objective

➢ Common objective for feedback control:

- ☞ **Case 1**: to maintain **CV** at its setpoint
- ☞ **Case 2**: to maintain **CV** in an acceptable range

➢ Eliminating **I** mode if possible:

- ☞ **I** mode: to eliminate offset or steady state error
- ☞ **I** mode is not required when allowing **CV** to vary within a range
- ☞ Eliminating **I** mode allows the use of *higher* proportional gain to reduce initial deviation of **CV** caused by disturbances

Situations Allowing CV to Vary In A Range

➢ **Case 2a**: *process is very controllable (large time constant + small dead time) that K_c can be set high and maintain CV in a very narrow range*

- ☞ Control of level in evaporators and reboilers
- ☞ Control of temperature in refrigeration systems, ovens, air conditioning/heating systems
- ☞ On-off control can be used when time constant is long enough that the cycling is of a very slow frequency
- ☞ Could use very narrow proportional band (high gain) for **P** or **PD** controllers or very narrow dead band for **on-off** controllers to maintain CV in a very narrow range
- ☞ D mode can be added to compensate for lag in sensor or final control element and thus improve stability

Situations Allowing CV to Vary In A Range

➢ **Case 2b**: *when it is desirable to allow CV to vary over a wide range*

- ☞ Control of level in intermediate storage tanks, condenser accumulators
- ☞ Control of pressure in gas surge tanks
- ☞ Use **P** controllers with as wide a proportional band as possible

Liquid Level and Gas Pressure Control

➢ Liquid level and gas pressure are controlled either for

☞ **Tight Control**:
To keep liquid level or gas pressure constant because of their effect on process or equipment operation

☞ **Averaging Control**:
To smooth out variations in flow while satisfying material balance

Liquid Level and Gas Pressure Control

Tight Control: Examples

- ➢ Level in natural circulation evaporators or reboilers
  - ☞ Too low a level causes deposits on bare hot tube
  - ☞ Too high a level causes elevation of boiling point
- ➢ Regulation of pressure in liquid or gas supply header keep constant P to prevent disturbances to users when there is a sudden change in demand of one or more of the users

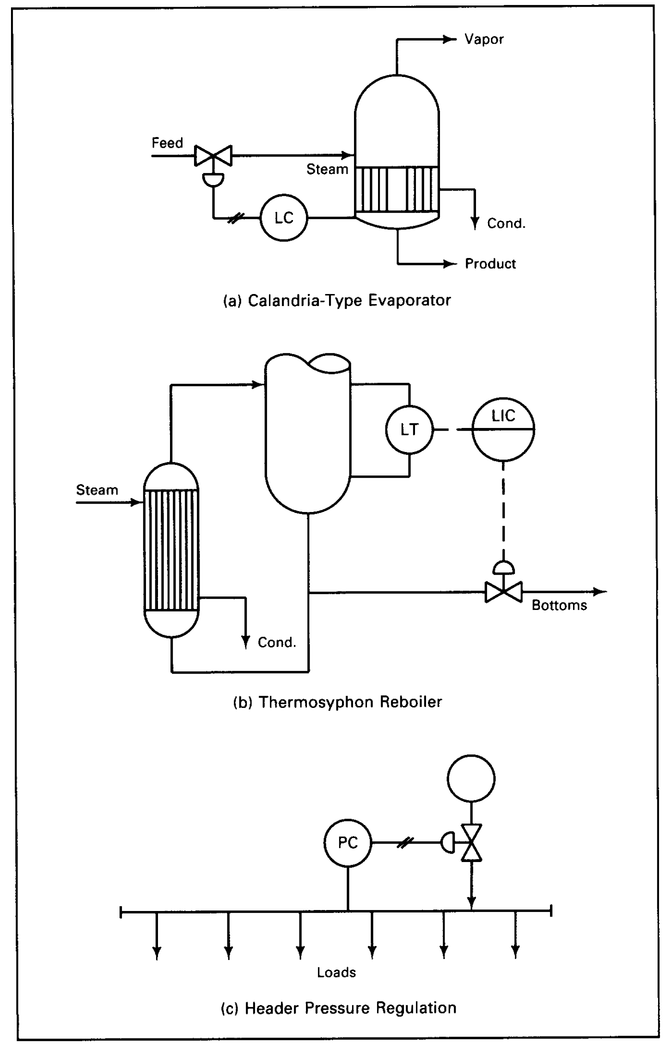


Fig. 5-1. Examples of Tight Control.

Liquid Level and Gas Pressure Control

Tight Control: Discussion

- Tight liquid level or gas pressure control systems require a fast-acting control valve with a positioner to avoid secondary time lag

  (Secondary time lag would make the loop less controllable and cause oscillatory behavior at high controller gain)

- If liquid level/gas pressure controller is cascaded to a flow controller, the latter must be tuned as tight as possible

- Tight controller needs only **P** action with gain set high $(10 \sim 100)$

- If lag of level/pressure sensor is significant
  $\Rightarrow$ use **D** mode and higher gain

- Derivative time: $\approx$ sensor time constant

- **I** mode should not be used
  (**I** mode would require a reduction of **P** gain)

Liquid Level and Gas Pressure Control

Averaging Control: Examples

- Level in a surge or intermediate storage tank
- Level in a condenser accumulator drum
  $\Rightarrow$ level has no effect on process operation
- Purpose of **averaging control**:
  to smooth out flow variations while keeping the tank from overflowing or running empty
- If **tight level control**
  $\Rightarrow$ outlet flow = inlet flow: a *pipe* ?

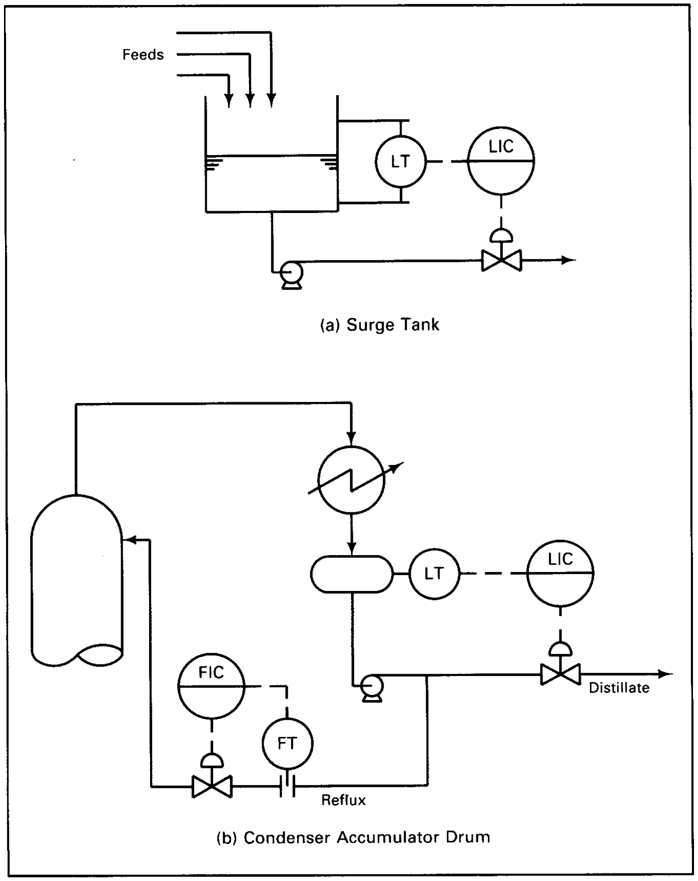


Fig. 5-2. Averaging Level Control.

Liquid Level and Gas Pressure Control

Averaging Control: Discussion

- **P action only**:
  setpoint = 50%, gain $= 1$, output bias = 50%
  using full capacity of the valve and of the tank:
  - Outlet valve fully opened when level is at 100% of range
  - Outlet valve fully closed when level is at 0% of range
- **Use a higher gain**:
  reduce the effective capacity of the tank for smoothing variations in flow
- **Use a lower gain**:
  reduce the effective capacity of the control valve and create possibility of tank overflowing or running dry

➢ Tank $\approx$ a low-pass filter to flow variations with time const.

$$\tau_f = \frac{A[h_{max} - h_{min}]}{K_c F_{max}} = \frac{1}{K_c} \frac{\Delta V}{F_{max}}$$

➢ **Increasing gain**
$\Rightarrow$ reducing time constant
less smoothing of flow variations

➢ **Doubling gain by 2** $=$
$\Rightarrow$ reducing tank area or transmitter range (by a factor of 2)

➢ **Reducing gain by 2** $=$
$\Rightarrow$ reducing valve capacity by half
$\Rightarrow$ increasing possibility of tank overflowing

➢ **Averaging pressure control**:
a simple fixed resistance on surge tank outlet is OK !

Flow Control

Controller Mode

- Use **PI control:** *weak* **P and** *strong* **I**
  proportional gain less than one
  very fast integral time ($\sim$ sec)
  (similar to a pure integral controller)

  - ☞ Flow process is very *simple* (= actuator)
  - ☞ Most flow sensors are *fast*:
    (orifice, venturi, flow tubes, magnetic flowmeters, turbine meters)
    $\Rightarrow$ most significant lag: valve actuator (a few seconds)
  - ☞ Simple process + fast sensor response $\Rightarrow$ very *stable*
  - ☞ *Noisy* measurement in flow sensors (turbulent flow)
  - ☞ Common control objective:
    maintain a *constant rate* with few manual changes in setpoint

Flow Control

Slave Flow Loop

- As a slave flow controller in a cascade control scheme:
  $\Rightarrow$ respond $fast$ to setpoint changes, $tight$ control
  $\Rightarrow$ **PI** controller with higher gain (> 1) and increasing integral time for stability

- Synthesis formula:
  - ☞ Reset time be set equal to time constant of the loop (that of control valve actuator)
  - ☞ Adjust gain for desired tightness of control

Flow Control

Valve with Hysteresis

- ➢ Caused by dynamic friction in the valve stem
- ➢ Causes variations in flow around its set point
- ➢ Creates a difference between actual valve position and controller output
- ➢ The error changes direction according to the direction that the stem must move
  $\Rightarrow$ Dead band around desired valve position
- ➢ Increasing controller gain
  $\Rightarrow$ reducing amplitude of flow variations !
- ➢ Valve positioners also reduce hysteresis and speed up valves

Temperature Control

- ➢ Significant lag in temperature sensors:
  $\Rightarrow$ **PID** controllers

- ➢ Synthesis method: $T_D =$ sensor time constant (τ_s)

$$\tau_s = \frac{MC_p}{hA} \qquad \text{(min)}$$

- ☞ M: mass of sensor, including thermowell (ℓb)
- ☞ C_p: specific heat (Btu/ℓb-^{o}F)
- ☞ h: film coefficient of heat transfer (Btu/min-ft^2-^{o}F)
- ☞ A: area of the thermowell (ft^2)

- When temperature controller manipulates flow of steam or fuel to a heater or furnace:
  - ☞ Heat of condensation of steam and heating value of fuel remain approximately constant with load
  - ⇒ rate of heat is proportional to the flow of steam or fuel

➢ When temperature controller manipulates flow of cooling water or hot oil:

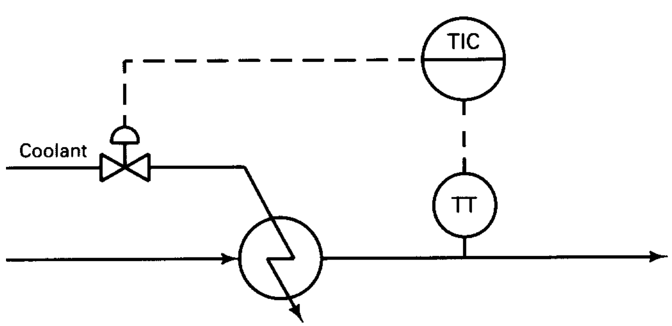
- ☞ Heat transfer rate requires that outlet utility temperature get closer to its inlet temperature as the heat transfer rate increases
- ☞ It requires higher increments in flow for equal increments in heat rate as load increases

  $\Rightarrow$ heat rate is *very nonlinear* with water or oil flow
- ☞ Because of this and other problems with excessive water temperature at low heat transfer rate

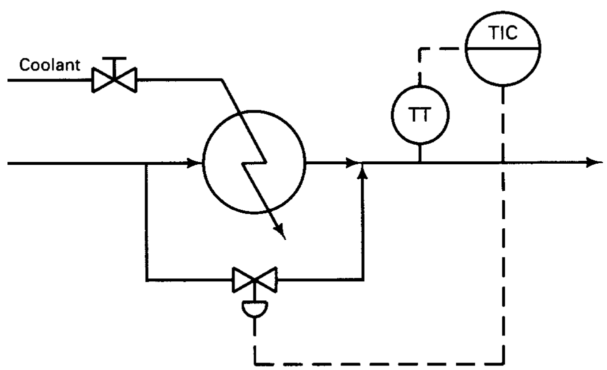
  $\Rightarrow$ heat exchanger with *bypass stream*

  $\Rightarrow$ *removes exchanger lag* from temperature control loop

  $\Rightarrow$ *faster* control



a. By Manipulation of Coolant Flow



b. By Manipulation of Bypass Stream

➢ Estimation of Sensor Time Constant

☞ RTD (Resistance Temperature Device):

weight: 0.5 ℓb; specific heat: 0.033 Btu/ℓb-^{o}F

☞ Thermowell:

cylindrical, outside diameter and length of 0.5 in.

☞ Film coefficient of heat transfer between fluid and thermowell:

90 Btu/h-ft^2-^{o}F

☞ Area of thermowell:

$3.1416(0.5)(5)/144 = 0.055$ ft^2

☞ Time constant:

$(0.5)(0.033)/(90/60)(0.055) = 0.20$ min

Analyzer Control

- **Major problems** associated with sensor/transmitter:
  - ☞ *Dead time* from sampling of process stream
  - ☞ *Measurement noise* due to poor mixing (sample is not representative)
  - ☞ Sensors are *slow*, *sensitive* to temperature and other process variables
  - ☞ *Discontinuous* in time usually
- The **important parameter**: ratio of dead time to process time constant

- **Case 1**:
  sampling time and total dead time
  $\approx$ process time constant

  ☞ Use **PID** controllers

  ☞ Any tuning formula could be used

  ☞ Synthesis formula have an advantage over the others

- **Case 2**:
  sampling time and total dead time
  $\gg$ process time constant

  ☞ Process is fast relative to the same frame in which it can be measured

  ☞ Situation is effectively the same as that of a fast process

  ☞ A pure **I** controller is suggested by controller synthesis

Typical Tuning Values for Particular Loops

TABLE 8-3: RULES OF THUMB FOR TUNING COMMON CONTROL LOOPS			
Loop Type	*Gain (PB)*	*Reset, Mins/Repeat (Repeats/Min)*	*Derivative, Minutes*
Flow	0.4–0.65 *(150%–250%)*	0.1–0.25 *(4–10)*	None
Temp.	2–10 *(10%–50%)*	2–10 *(0.1–0.5)*	0.1–2 (always less than reset)
Pressure, Gas	20–50 *(2%–5%)*	May not be needed	None
Pressure, Liquid	0.5–2.0 *(50%–200%)*	0.1–0.25 *(4–10)*	None
Pressure, Vapor	2–10 *(10%–50%)*	2–10 *(0.1–0.5)*	0.1–2 (Always less than reset)
Level	2–20 *(5%–50%)*	1–5 *(0.2–1.0)*	None
Composition	0.1–1.0 *(100%–1000%)*	10–30 *(0.03–0.1)*	Varies

Practical Controller Tuning Tips

Practical Controller Tuning Tips

- ➢ For making efficient and satisfying controller tuning
- ➢ Controller tuning need only be approximate rather than precise

Practical Controller Tuning Tips

Tune Coarse, Not Fine

- Performance of a PID controller is $NOT\ sensitive$ to precise adjustment of its tuning parameters

- There is satisfaction in the large improvements in performance achievable by coarse tuning
  $\Rightarrow$ frustration in improving performance by fine tuning

- Controller tuning need only be approximate rether than precise

- An expert seldom increases a parameter to less than twice or decreases it by less than half its current value

Practical Controller Tuning Tips

Tune with Confidence

- Any of parameters may be adjusted to make up for non-optimal values of the other parameters

- A **successful approach** is:
  - Select T_I first
  - Set T_D to about one fourth of T_I
  - Adjust K_c to obtain tight control of C.V. without undue variations in the M.V.
  - If response is too oscillatory
    $\Rightarrow$ double T_I, T_D , re-adjust K_c
  - If response is too slow in approaching set point
    $\Rightarrow$ halve T_I, T_D , re-adjust K_c
  - When satisfactory performance is obtained
    leave it alone, do NOT try to fine tune it further
  - Fine tuning it will result in disappointment
    because of the insignificant incremental improvement

Practical Controller Tuning Tips

Use All of Available Information

- Enough information about the process equipment may be gathered to estimate process gain, time constant, dead time *without* having to resort to open-loop test

- Information can also be gathered during trial-and-error tuning

- Estimating T_I, T_D from period of oscillation or total dalay $(\tau + d)$ around the loop

- $\tau + d$: can be estimated by time difference between peaks in controller output (or transmitter signal)

Practical Controller Tuning Tips

Try A Longer Integral Time

- Poor loop response can many times be traced to trying to bring the **CV** back to its **SP** *faster* than the process can respond
  $\Rightarrow$ Increasing T_I
  $\Rightarrow$ Increasing controller gain, improving response

Practical Controller Tuning Tips

Tuning Very Controllable Processes: $\frac{d}{\tau} < 0.1$

- Having very large ultimate gains
- Difficult to determine K_U, T_U by *ultimate gain-period method*
- M.E.I. formula give very high gains and very fast reset time
- Previous tuning formula: not suitable for $\frac{d}{\tau} < 0.1$
  $\Rightarrow$ to let good judgment override the formula

Practical Controller Tuning Tips

Tuning Very Uncontrollable Processes: $\frac{d}{\tau} > 1.0$

➢ Even optimally tuned feedback controller will result in poor performance:

☞ Large initial deviations on disturbance inputs

☞ Slow return to set point change

➢ Improved performance can be achieved through *feedforward control*, *dead time compensation* in feedback controller

Practical Controller Tuning Tips

Beware of Problems Not Related to Tuning

- Reset windup, caused by saturation of controller output
- Interaction between loops ($\Rightarrow$ *decoupling*)
- Processes with inverse or overshoot response, caused by parallel effects of opposite direction between a process input and C.V.
- Changes in process parameters because of nonlinearities
  $\Rightarrow$ adaptive control

Integrated Tuning and Diagnosis

Integrated Tuning and Diagnosis

➢ **Problems Out of Tuning:** Poor Control Performance

- ☞ Poor controller tuning
- ☞ Bad control configuration
- ☞ Nonlinearities in the valve (stick, hysteresis $\cdots$)
- ☞ Improperly sized valves and transmitters

➢ **Important:** to discover problems before initializing controller tuning

Friction in the Valve

- Too large **static friction** (stiction) in valve
  $\Rightarrow$ degraded control performance

- To measure amount of friction:
  making small change in control signal and
  checking process output (or valve stem)

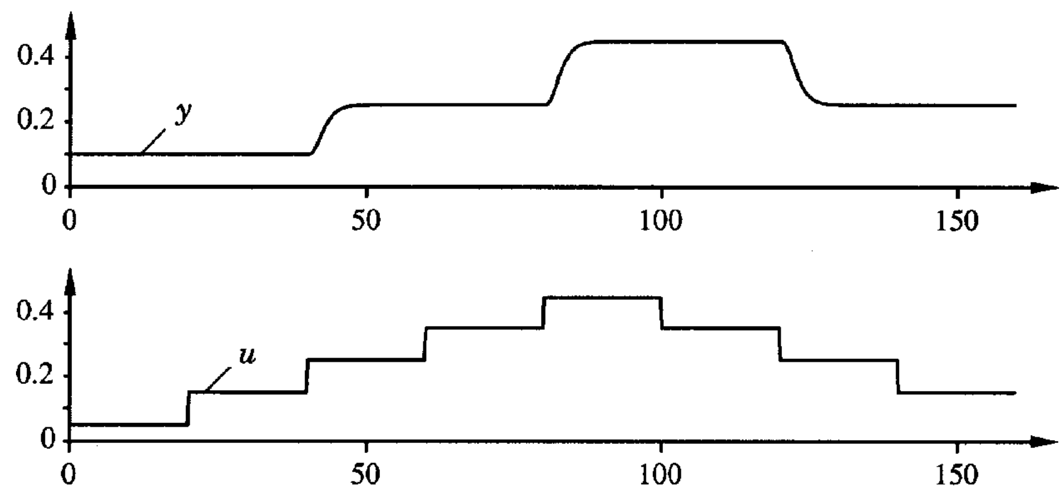


Figure 6.10 Procedure to check the amount of valve friction. The upper diagram shows process output y and the lower diagram shows control signal u.

➢ **Static friction** in valve

⇒ Stick-slip motion

⇒ Process output oscillates around setpoint (control ≈ triangular; measurement ≈ square)

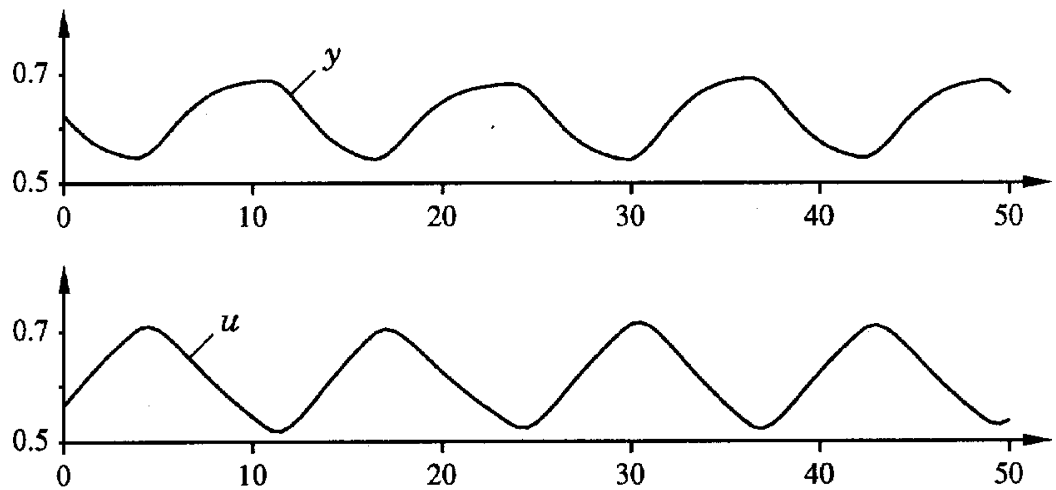


Figure 6.11 Stick-slip motion caused by friction in the valve. The upper diagram shows process output y and the lower diagram shows control signal u.

Diagnosis for Oscillation

- **Oscillation**

  ⇒ De-tune PID (many operators will do this)

  ⇒ Oscillation with larger period !

- To determine the cause of oscillation:

  ☞ **Manual control:**

  check if oscillations are generated inside or outside the loop

  ☞ **Check friction:**

  making small changes in control signal

  checking if measurement signal follows

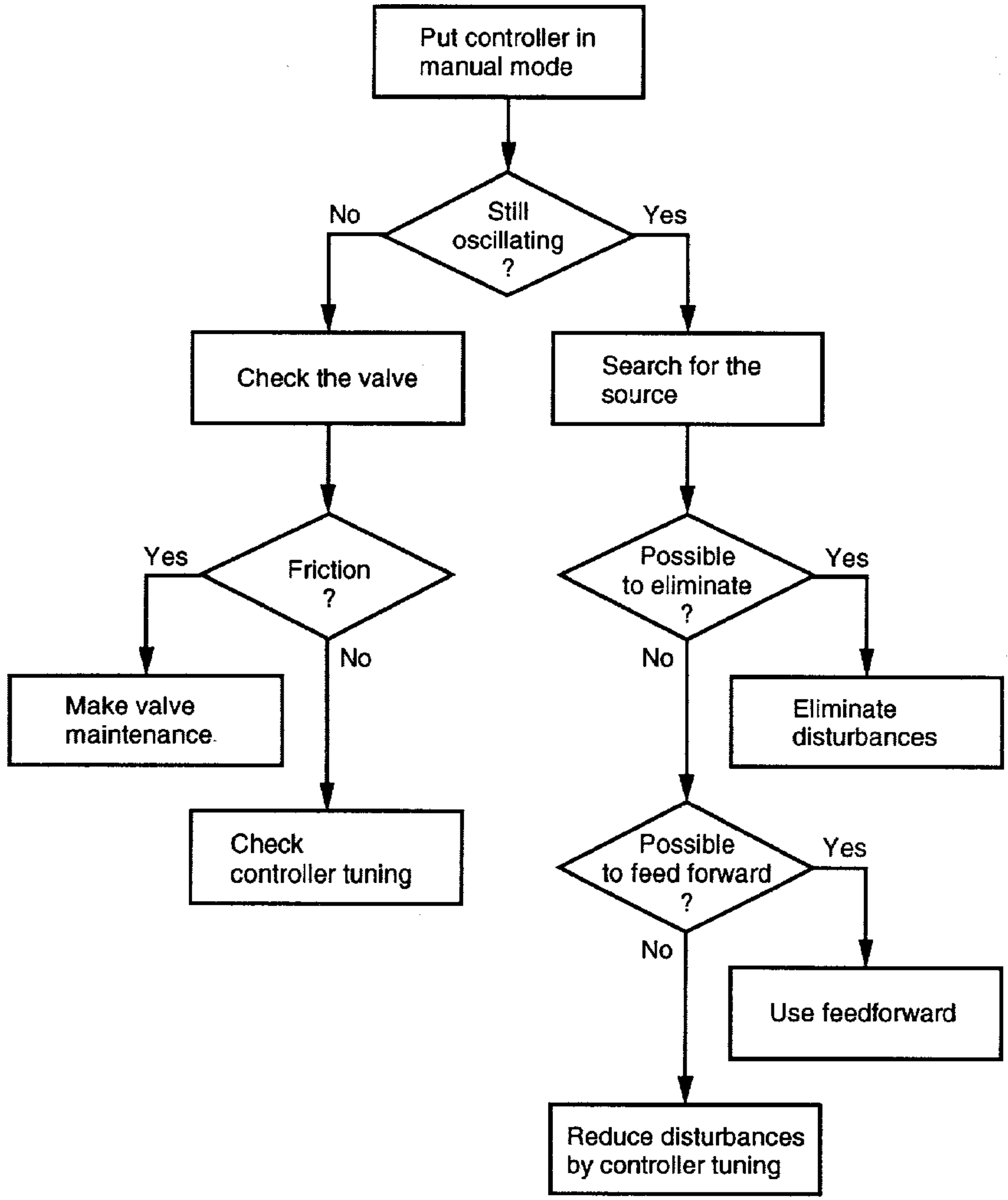


Figure 6.12 Diagnosis procedure to discover the cause of oscillations, and recommended actions to eliminate them.

Hysteresis in the Valve

- ➢ Wear $\Rightarrow$ **hysteresis** (backlash) in valve or actuator
- ➢ Measurement of amount of **hysteresis** (I):
  - ☞ Two step changes in control signal (same direction)
  - ☞ 3rd step in opposite direction (same size to 2nd step)
  - ☞ **Hysteresis** $= \frac{\Delta y}{K_p}$, K_p: process gain, $\Delta y = y_3 - y_1$

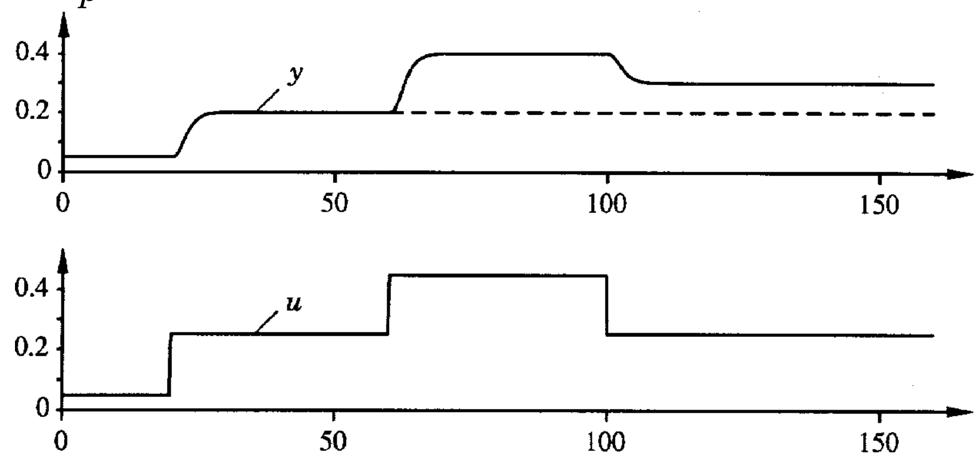


Figure 6.13 Procedure to check the amount of valve hysteresis. The upper diagram shows process output y and the lower diagram shows control signal u.

➢ Measurement of amount of **hysteresis** (II):

- ☞ Ramped control signal upwards and downwards
- ☞ **Hysteresis**: horizontal distance between two lines

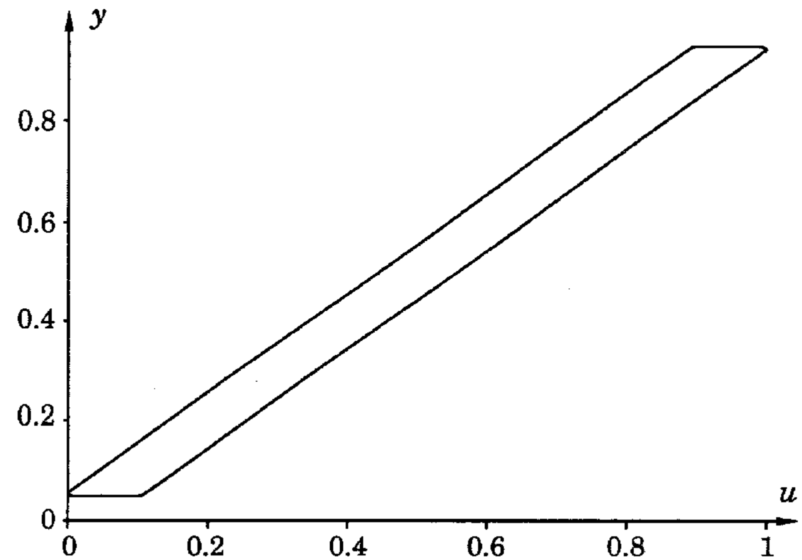


Figure 6.14 Characteristic of a valve with hysteresis. The diagram shows process output y as function of control signal u.

➢ Closed-loop with large **hysteresis**: linear drift

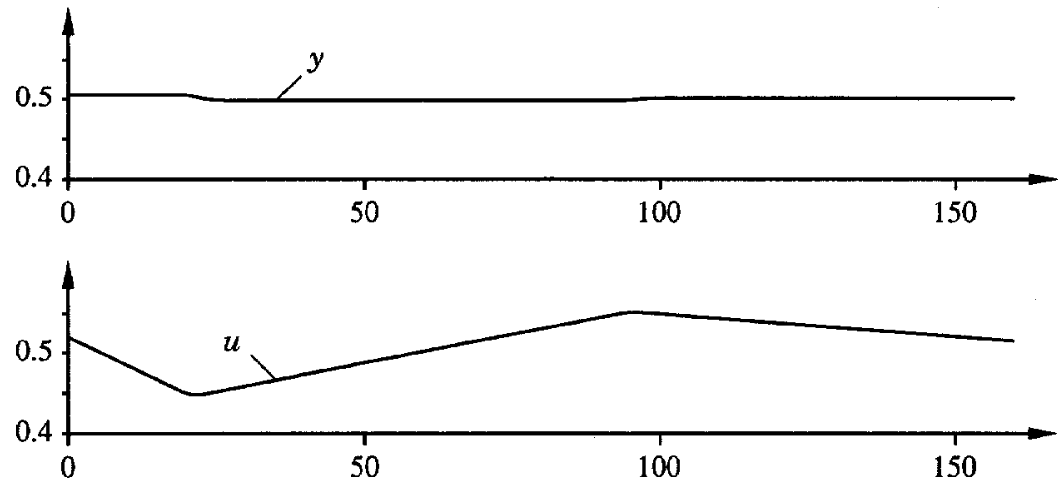


Figure 6.15 Closed-loop control with valve hysteresis. The upper diagram shows process output y and the lower diagram shows control signal u.

Other Nonlinearities

➢ Even valves with a small **static friction** and **hysteresis** often have a nonlinear characteristics (sensor, transmitter $\cdots$)

➢ Total characteristic of process can be obtained by checking static relation between control signal and measured signal

➢ **Ex:** larger gain at larger valve positions
$\Rightarrow$ gain scheduling ?

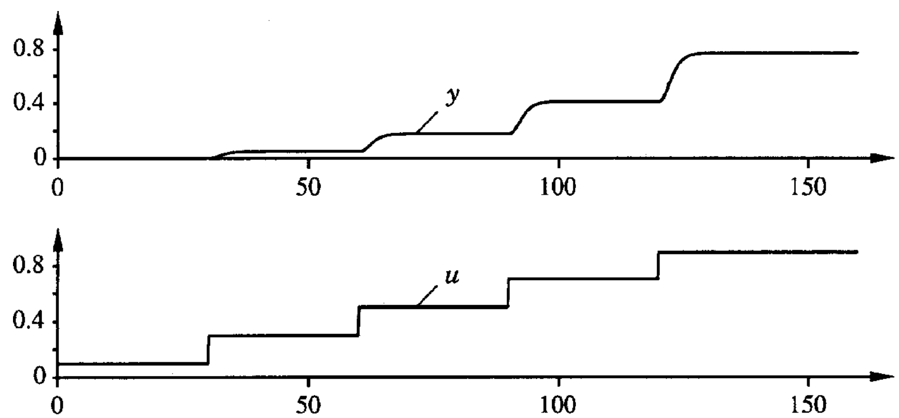


Figure 6.16 A procedure to determine the static process characteristic. The upper diagram shows process output y and the lower diagram shows control signal u.

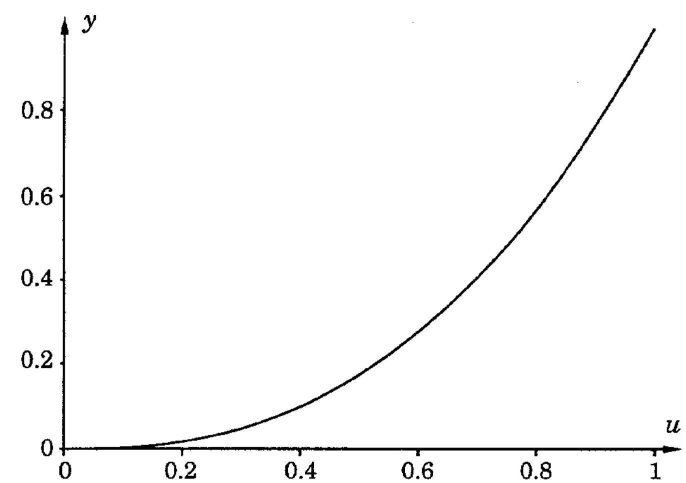


Figure 6.17 The static process characteristic, showing process output y as function of control signal u.

Integrated Tuning and Diagnosis

- On-line detection methods (?) are important to continuous adaptation

- **On-line detection:**
  - Monitor control performance
  - Give alarm if process dynamics change

- **Adaptive control:**
  - Monitor control performance
  - Change controller parameters if process dynamics change

- It is important to determine **why** the performance has changed before actions are taken

- Lack of on-line detection methods in adaptive controller is perhaps the major reason for relatively few applications of continuous adaptive control available today

Tuning for Quarter Decay Ratio

Stability of Feedback Loop

➢ **Loop is unstable** when a small change in disturbance or set point causes the system to deviate widely from its normal operating point

➢ **Causes of Instability**

☞ **Controller has the incorrect action**
instability is manifested by controller output *running away* to either its upper or lower limit

☞ **Controller is tuned too tightly**
K_c too high ?, T_I too small ? T_D too high ?

☞ Process is inherently unstable (*rare*)

- **Oscillatory type of instability** is caused by the controller having
  - ☞ A too high gain, or
  - ☞ A too fast integral time, or
  - ☞ A too high derivative time

- We need a simple method for determining the
  ultimate gain and **period of oscillation**
  (The process starts to become oscillation)

Determination of Ultimate Gain/Period

➢ **Ultimate Gain**: K_{cu}

☞ K_{cu}: the gain of a P controller at which the loop oscillates with constant amplitude

☞ A measure of the controllability of the loop (larger **Ultimate Gain** $\longrightarrow$ easier loop)

☞ K_{cu}: gain at which loop is at threshold of instability

$K_c < K_{cu}$: stable; $K_c > K_{cu}$: unstable

➢ **Ultimate Period**: T_U (period of oscillations) a measure of speed of response of the loop (longer period $\longrightarrow$ slower loop)

➢ Procedure for determining K_{cu} and T_U:

- ☞ Remove integral and derivative modes
- ☞ Carefully increase K_c in steps
- ☞ Disturb loop (small step setpoint change)
  observe response of CV and MV
- ☞ Constant amplitude of oscillations: $K_c \equiv K_{cu}$
  average period of oscillation: T_U

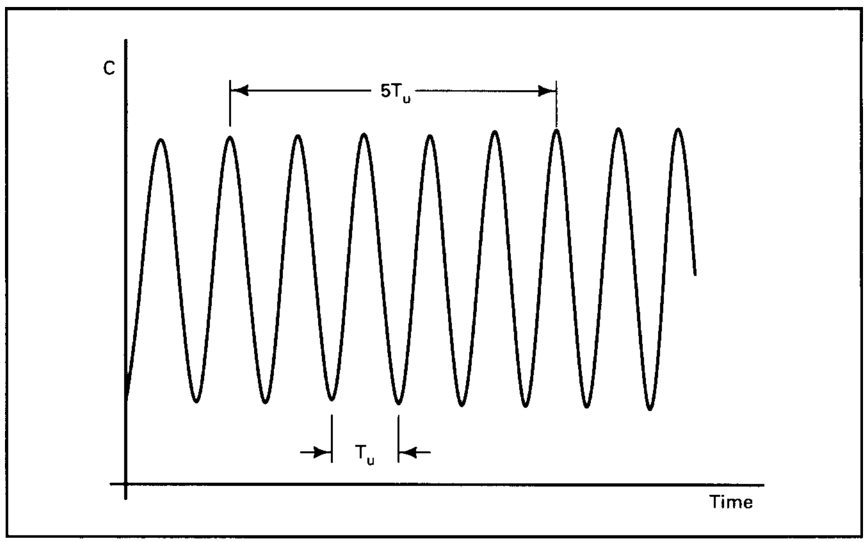


Fig. 2-13. Determination of Ultimate Period.

Tuning for Quarter-Decay Ratio (QDR) Response

Ziegler-Nichols (1942)

Controller	K_c	T_I	T_D
P	$0.50K_{cu}$	-	-
PI	$0.45K_{cu}$	$T_U/1.2$	-
PID series	$0.60K_{cu}$	$T_U/2.0$	$T_U/8$
PID parallel	$0.75K_{cu}$	$T_U/1.6$	$T_U/10$

➢ Note:

- ☞ Additional lag introduced by integral mode
  ⇒ a reduction of 10% in QDR gain from P to PI
- ☞ Derivative mode increases controlability of the loop
  ⇒ allows increasing 20% gain from P to PID
- ☞ $T_I = 4T_D$ in series PID
- ☞ Tuning parameters for QDR are NOT unique

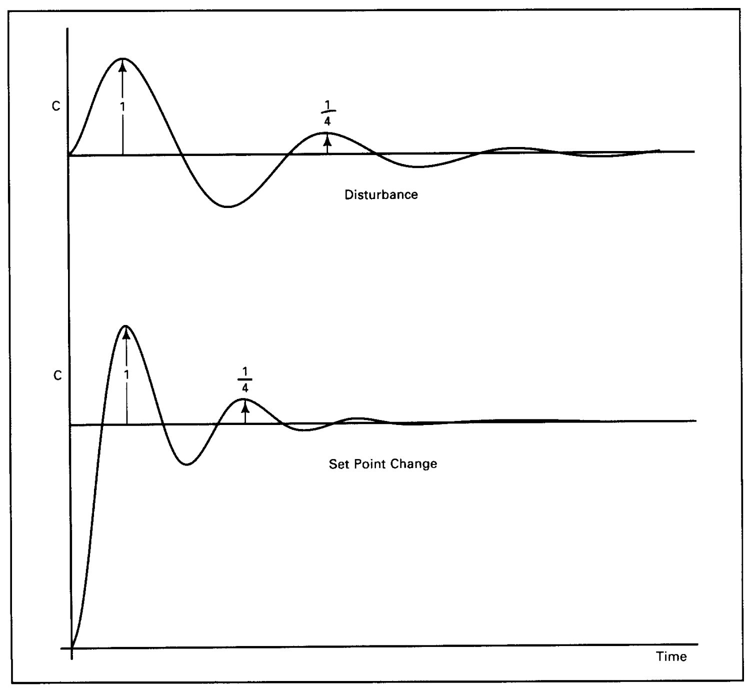


Fig. 2-14. Quarter-Decay Ratio (QDR) Responses.

➢ Example: Ultimate Gain Tuning of A Steam Heater

☞ A 2^oC change in set point is used to start the oscillations

☞ Try $K_c = 8,\ 12$

$$\Longrightarrow \quad K_{cu} = 12\%/\%\ (8.33\% PB), \quad T_U = 0.60 min$$

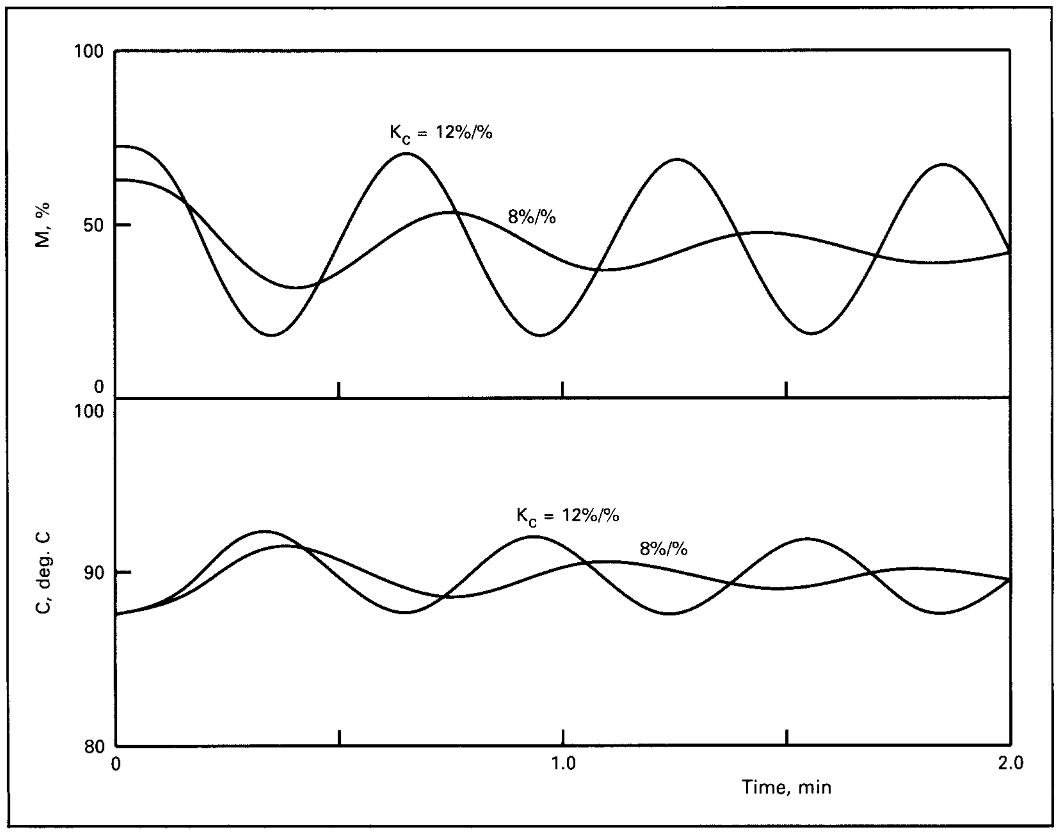


Fig. 2-15. Determination of Ultimate Gain and Period for Temperature Control Loop on Steam Heater.

$$
\begin{array}{lrcl}
\text{P control:} & K_c & = & 0.50 \cdot 12 = 6.0\ \%/\%\ (17\%\text{PB}) \\
\text{PI control:} & K_c & = & 0.45 \cdot 12 = 5.4\ \%/\%\ (18\%\text{PB}) \\
& T_I & = & 0.6/1.2 = 0.50\ \text{min} \\
\text{PID control:} & K_c & = & 0.60 \cdot 12 = 7.2\ \%/\%\ (14\%\text{PB}) \\
& T_I' & = & 0.6/2 = 0.30\ \text{min} \\
& T_D' & = & 0.6/8 = 0.075\ \text{min}
\end{array}
$$

➢ Response of Ultimate Gain Tuning of A Steam Heater

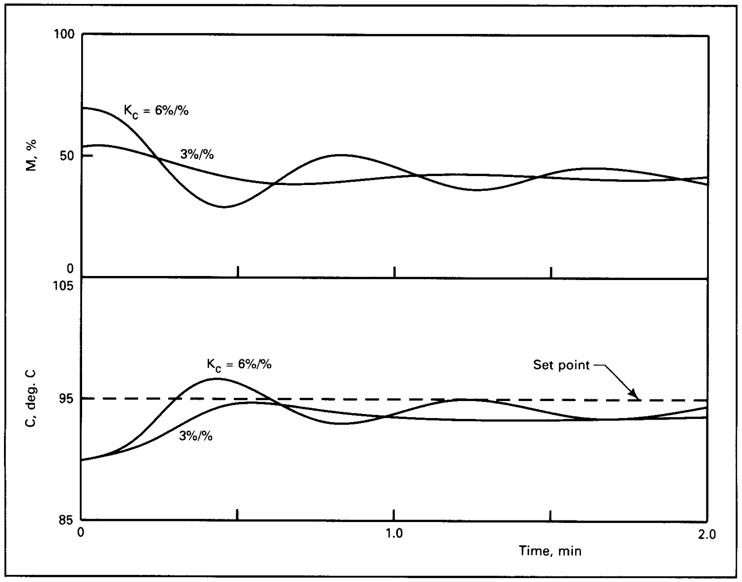


Fig. 2-16. Proportional Controller Response to a 5°C Change in Set Point.

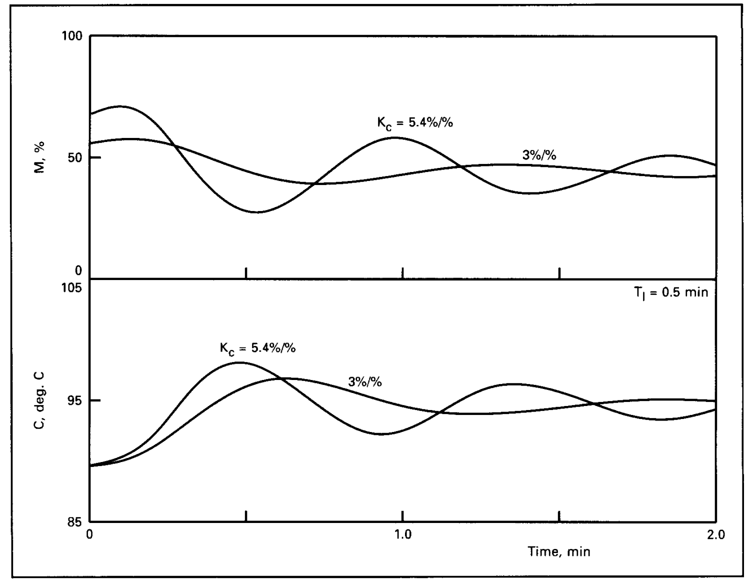


Fig. 2-17. Proportional-Integral (PI) Controller Response to a 5°C Change in Set Point (T_I = 0.5 minute).

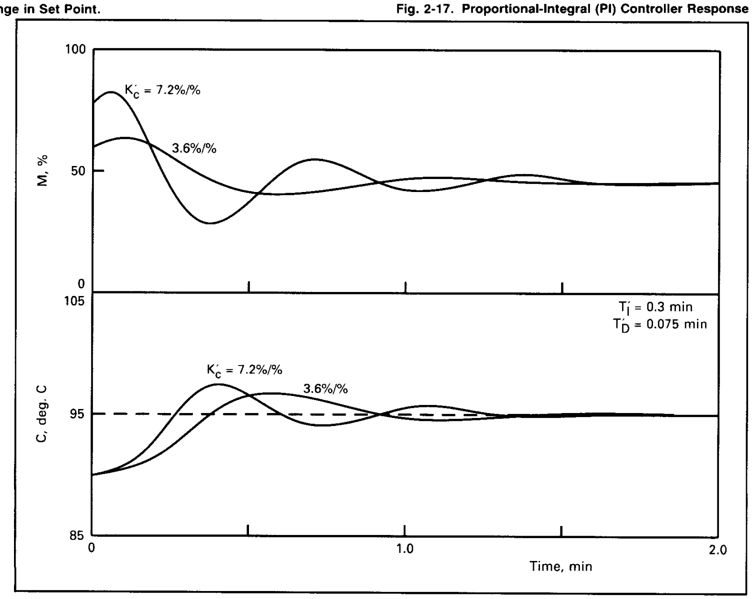


Fig. 2-18. Series PID Controller Response to a 5°C Change in Set Point (T_I = 0.30 minute; T_D = 0.075 minute).

Practical Ultimate Tuning Tips

- It is NOT absolutely necessary to force the loop to oscillate with constant amplitude in determining K_{cu}, T_U
  - T_U does not vary drastically as loop approaches ultimate condition
  - Any oscillation that would allow a rough estimate of T_U gives good enough values of T_I and T_D
  - K_c can be adjusted to obtain an acceptable response
  - **Example:** steam heater on last page

$$K_c = 8\%/\% \rightarrow \hat{T}_U = 0.7 \text{ min } (15\% \text{ off from } T_U)$$

- Performance of feedback controller is NOT usually sensitive to tuning parameters

  $\Rightarrow$ (a waste of time to change tuning par.s by less than 50%)

- Recommended parameter adjustment policy is to leave T_I **and** T_D **fixed** at values calculated from tuning formula and **adjust** K_c to obtain desired response

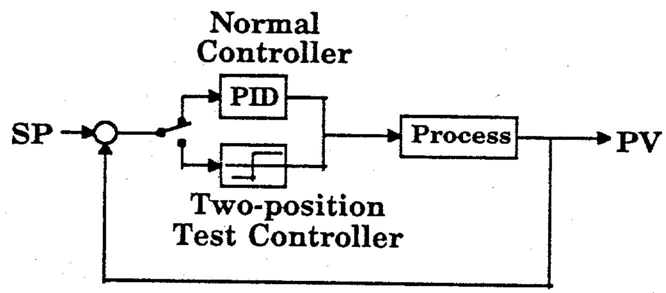
Other Methods to Determine Ultimate Gain and Ultimate Period

➢ An Approximate Experiment

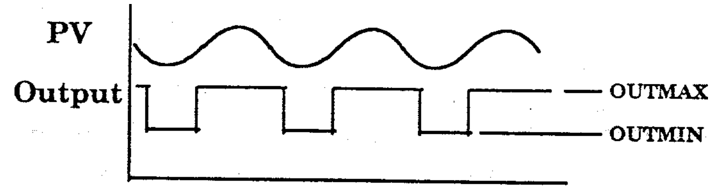
- ☞ Start out like you are going to make a closed-loop test
- ☞ Increase controller gain until you get 1/4 decay closed-loop response
  - ➠ K_{cq}: corresponding controller gain
  - ➠ T_q: period of 1/4 decay
- ☞ Estimate of K_{cu}, T_U:

$$
\begin{aligned}
K_{cu} &\approx \frac{5}{3}K_{cq} \;=\; 1.67K_{cq} \\
T_U &\approx 0.9T_q
\end{aligned}
$$

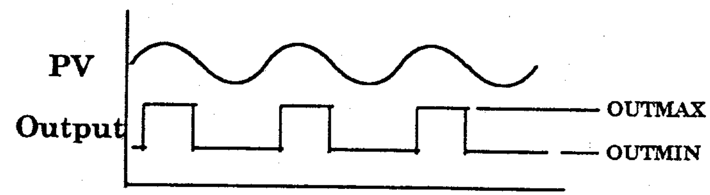
➢ Relay Method



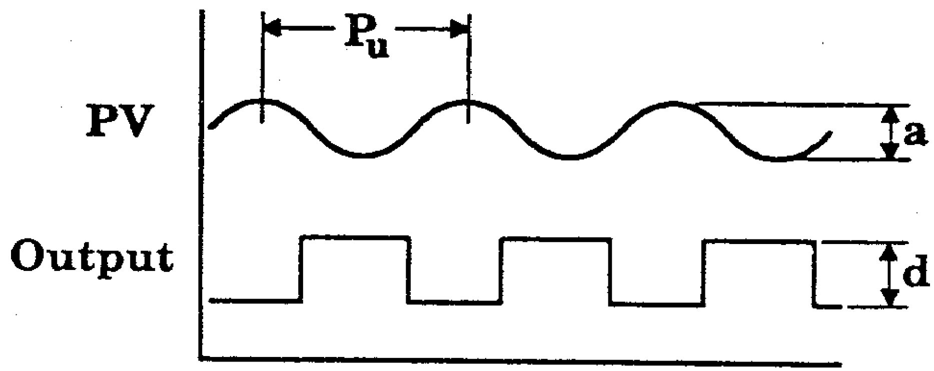
Set the controller output to operate between OUTMAX and OUTMIN



Increase both OUTMAX and OUTMIN by the same amount



Decrease both OUTMAX and OUTMIN by the same amount



$$K_{cu} = \frac{4d}{\pi a} \approx \frac{5d}{4a}$$

Need for Alternatives to Ultimate Gain Tuning

- It is not always possible to determine K_{cu} and T_U of a loop
- K_{cu} and T_U do NOT give insight into which process or control system characteristics could be modified to improve performance

- ➢ More fundamental methods of **characterizing process dynamics** are needed to guide such modifications
  - ☞ Open-loop reaction-curve methods
  - ☞ Closed-loop reaction-curve methods
- ➢ There is also the need to develop **tuning formula** for response other than **QDR** response
  - ☞ Optimization methods
  - ☞ Controller synthesis
  - ☞ Internal Model Control $\cdots$
  - ☞ Dominant pole placement

Open-Loop Testing of Process Dynamics

➢ **Purpose** of open-loop test: to determine process TF

TF is a more fundamental model than ultimate gain and period

➢ Two signals of interest:

Controller output ($u(t)$),

Transmitter output ($y(t)$) ($0\% \sim 100\%$)

$$
\begin{aligned}
G(s) &= \frac{Y(s)}{U(s)} \\
Y(s) &= \mathcal{L}\{y(t) - \bar{y}\} \\
U(s) &= \mathcal{L}\{u(t) - \bar{u}\}
\end{aligned}
$$

➢ **Procedure** for performing an **open-loop test**:

- ☞ Keep steady-state (automatic or manual control): $u = \bar{u}, \quad y = \bar{y}$
- ☞ Switch into manual control
- ☞ Cause a step change: $u(t) = \bar{u} + A$
- ☞ Record transmitter output $y(t)$ until new steady-state is reached
- ☞ Analysis of recorded data, obtaining parameters in $G(s)$

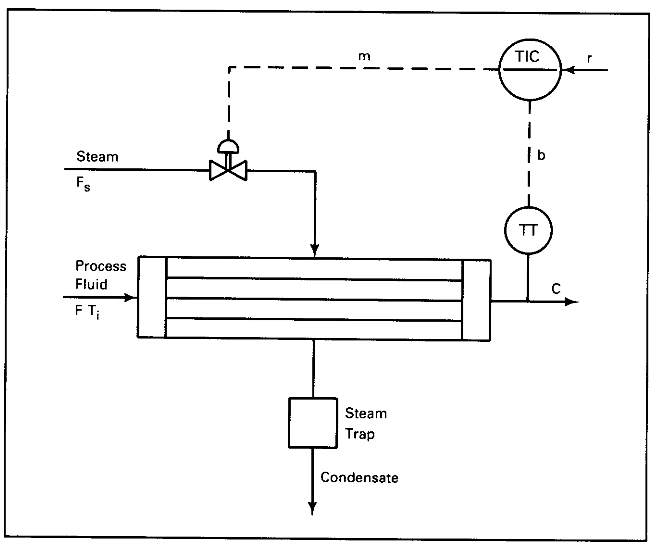
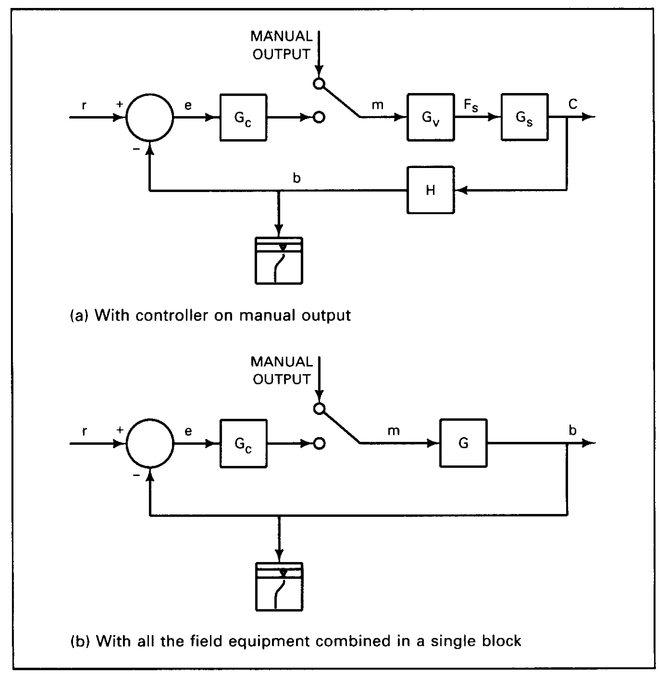


Fig. 3-1. Sketch of Temperature Control of Steam Heater.



(a) With controller on manual output

(b) With all the field equipment combined in a single block

Fig. 3-2. Block Diagram of Feedback Control Loop with Controller on Manual.

Two Parameters Model from Open-Loop Step Test

- ➢ Use **2 parameters** to characterize process dynamics
  - ☞ Parameter d: how long
  - ☞ Parameter a: how fast (?)
- ➢ **Advantage**: one simple test
  Disadvantage: vague meaning of a

➢ QDR Tuning Based on 2-Parameters Model

	gain	integral time	derivative time
P	$K_c = \frac{1}{a}$	-	-
PI	$K_c = 0.9\ \frac{1}{a}$	$T_I = 3.33\ d$	-
PID series	$K_c^{'} = 1.2\ \frac{1}{a}$	$T_I^{'} = 2.0\ \ d$	$T_D^{'} = 0.5\ d$

First Order Plus Dead-Time Model

- **3-parameters FOPDT model:** $G(s) = \dfrac{Ke^{-ds}}{\tau s + 1}$
  - ☞ d: **how long** it takes for the controller to detect the beginning of change in transmitter output
  - ☞ τ: **how fast** the controlled variable changes
  - ☞ K: **how much** the CV changes for a given change in CO

➢ QDR Tuning Based on FOPDT Model

☞ FOPDT model: $\dfrac{Ke^{-ds}}{\tau s+1}$

☞ Note: slope $= \dfrac{Aa}{d} = \dfrac{AK}{\tau} \Longrightarrow a = K\dfrac{d}{\tau}$

	gain	integral time	derivative time
P	$K_c = \dfrac{\tau}{K\ d}$	-	-
PI	$K_c = 0.9\ \dfrac{\tau}{K\ d}$	$T_I = 3.33\ d$	-
PID series	$K_c^{'} = 1.2\ \dfrac{\tau}{K\ d}$	$T_I^{'} = 2.0\ \ d$	$T_D^{'} = 0.5\ d$

FOPDT Model from Step Response

- **FOPDT** model: $\dfrac{\Delta y(s)}{\Delta u(s)} = G(s) = \dfrac{Ke^{-ds}}{\tau s + 1}$

$$\tau \frac{d\Delta y(t)}{dt} + \Delta y(t) = K\Delta u(t-d)$$

or $$\tau \frac{dY(t)}{dt} + Y(t) = KU(t-d)$$

➢ Step response and some characteristics:

$$\begin{aligned} \Delta u(t) &= A \qquad\qquad (\text{for} \quad t \geq 0) \\ \Delta y(t) &= AK\left[1 - e^{(t-d)/\tau}\right] \end{aligned}$$

$$\begin{aligned} \Delta y(t=\infty) &= A\ K \\ \Delta y(t = d + \tfrac{\tau}{3}) &= 0.283\ KA \\ \Delta y(t = d + 0.4\tau) &= \tfrac{1}{3}\ KA \\ \Delta y(t = d + \tau) &= 0.632\ KA \\ \Delta y(t = d + 1.1\tau) &= \tfrac{2}{3}\ KA \\ \frac{d\Delta y(t=d)}{dt} &= \frac{KA}{\tau} \quad = \quad \left.\frac{d\Delta y(t)}{dt}\right|_{max} \end{aligned}$$

➢ FOPDT Model: Process Gain from Step Test

☞ Steady-state gain K for a self-regulating process

$$K = \left.\frac{\Delta y}{\Delta u}\right|_{ss} = \frac{\%\text{transmitter output}, \%\text{TO}}{\%\text{controller output}, \%\text{CO}}$$

☞ **Example**: step test on steam heater

$$\Delta u = 0.8\ mA \times \frac{(100-0)\%\text{CO}}{(20-4)\ mA} = 5\ \%\text{CO}$$

$$\Delta y = 5^oC \times \frac{(100-0)\%\text{TO}}{(150-50)^oC} = 5\ \%\text{TO}$$

$$\Rightarrow K = \frac{5\%\text{TO}}{5\%\text{CO}} = 1.0\ \frac{\%\text{TO}}{\%\text{CO}}$$

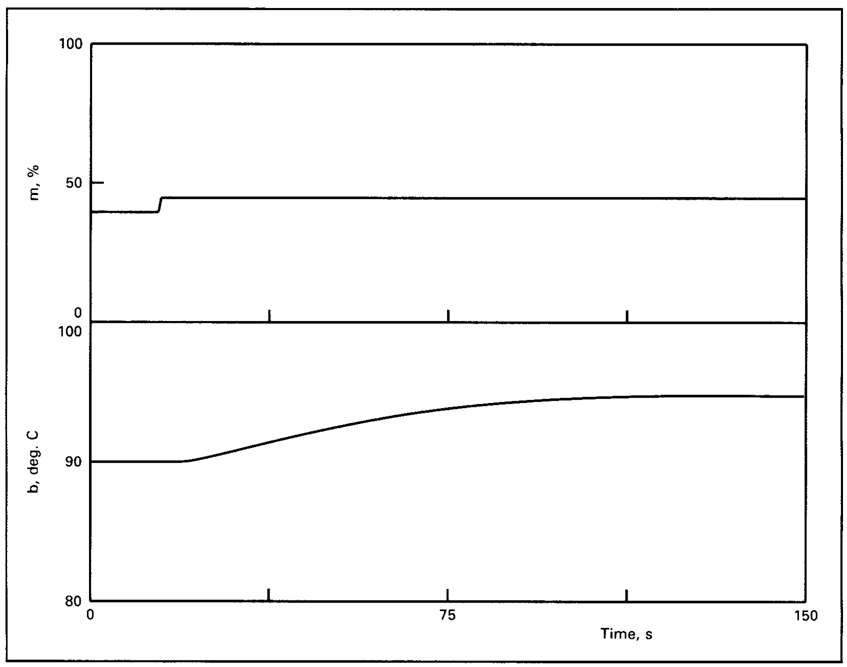


Fig. 3-3. Response for Step Test of Steam Heater.

➢ FOPDT Model: Time Constant and Dead Time Tangent Method (fit 1)

☞ Find point with *maximum slope*, take tangent line

☞ Tangent line crosses *initial* steady state ($\bar{y}$): $t_1 = d$

☞ Tangent line crosses *new* steady state ($\bar{y} + KA$): $t_2 = d + \tau$

$$\Rightarrow \quad d = t_1; \qquad \tau = t_2 - t_1$$

☞ **Problems:**

➡ Tangent line is not very reproducible

➡ Larger estimate of time constant → tighter controller tuning

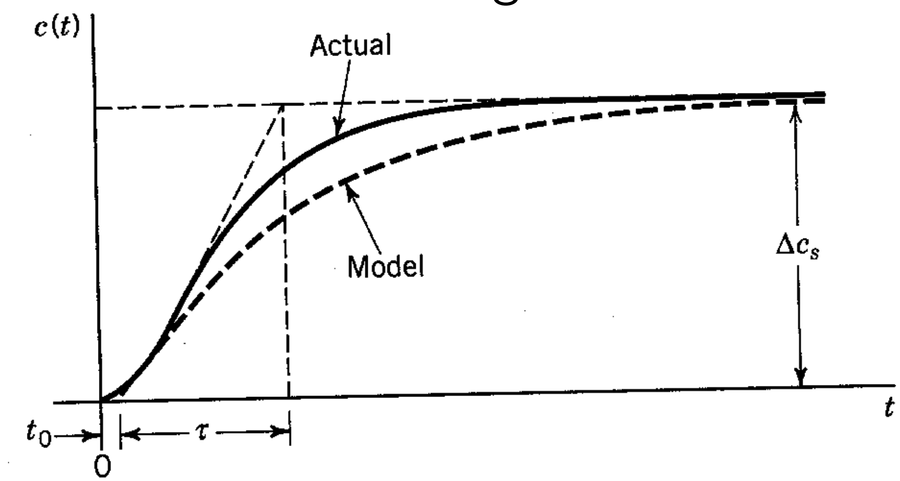


Figure 7-2.6a FOPDT model parameters by fit 1.

➢ FOPDT Model: Time Constant and Dead Time Tangent and Point Method (fit 2)

- ☞ Find point with *maximum slope*, take tangent line
- ☞ Tangent line crosses *initial* steady state ($\bar{y}$): $t_1 = d$
- ☞ Response reaches 63.2% of total steady-state change:

$$t_{0.632} = d + \tau$$

$$\Rightarrow \quad d = t_1; \qquad \tau = t_{0.632} - t_1$$

- ☞ **Problems:**
  - ➠ Tangent line is not very reproducible
  - ➠ Shorter estimate of τ $\rightarrow$ conservative tuning

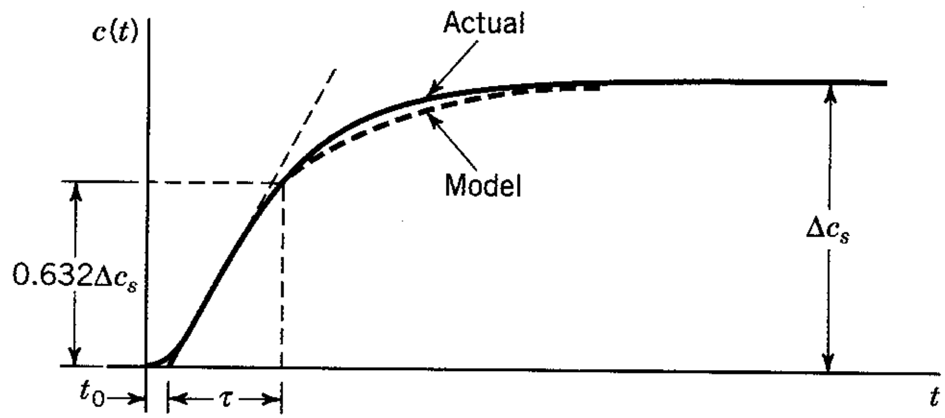


Figure 7-2.6*b* FOPDT model parameters by fit 2.

➢ FOPDT Model: Time Constant and Dead Time Two-Point Method (fit 3)

☞ Step response reaches 28.3% of SS change: $t_{0.283} = d + \frac{\tau}{3}$

☞ Step response reaches 63.2% of SS change: $t_{0.632} = d + \tau$

$$\Rightarrow \quad \tau = 1.5\left(t_{0.632} - t_{0.283}\right) \qquad d = t_{0.632} - \tau$$

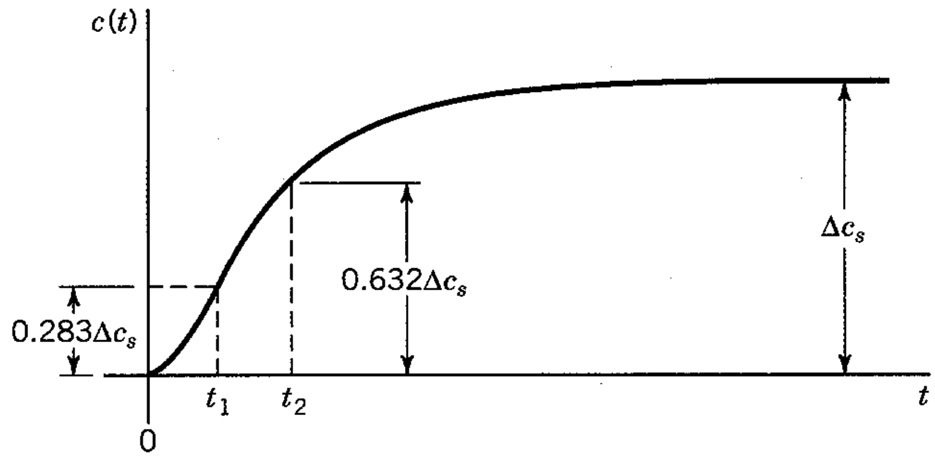


Figure 7-2.6c FOPDT model parameters by fit 3.

which reduces to

➢ FOPDT Model: Time Constant and Dead Time Two-Point Method (fit 4)

☞ Step response reaches $1/3$ of SS change: $t_{1/3} = d + 0.4\tau$

☞ Step response reaches $2/3$ of SS change: $t_{0.632} = d + 1.1\tau$

$$\Rightarrow \quad \tau = 1.4(t_{2/3} - t_{1/3}) \qquad d = t_{2/3} - 1.1\tau$$

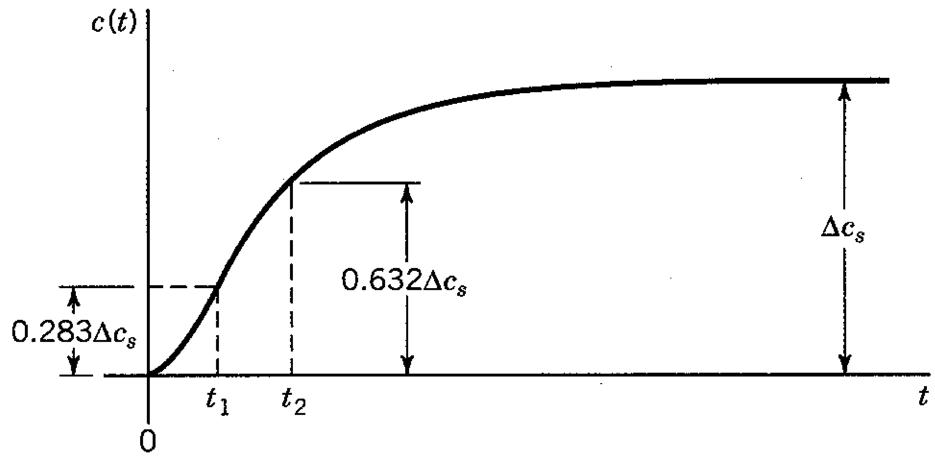


Figure 7-2.6c FOPDT model parameters by fit 3.

which reduces to

☞ **Advantage:** more reproducible

☞ **Problem:** longer estimate of d, shorter estimate of τ

➢ Open-Loop Testing: Steam Heater

☞ **Step response**: figure 3.4

☞ **Tangent method**:

$$\begin{aligned} d &= & 8.0\ sec &= 0.13\ min \\ \tau &= 57.2 - 8.0 &= 49.2\ sec &= 0.82\ min \end{aligned}$$

☞ **Tangent-and-point method**:

$$\begin{aligned} d &= & 8.0\ sec &= 0.13\ min \\ T_{63.2\%} &= 90 + 5 \times 0.632 &= 93.2^oC & \\ t_{63.2\%} &= 45\ sec & & \\ \tau &= 45 - 8.0 &= 37\ sec &= 0.62\ min \end{aligned}$$

☞ Two-point method:

$$
\begin{aligned}
T_{28.3\%} &= 90 + 5 \times 0.283 &&= 91.4^oC \\
T_{63.2\%} &= 90 + 5 \times 0.632 &&= 93.2^oC \\
t_{28.3\%} &= 23\ sec \quad t_{63.2\%} &&= 45\ sec \\
\tau &= 1.5(45 - 23) &&= 33\ sec &&= 0.55\ min \\
d &= 45 - 33 &&= 12\ sec &&= 0.2\ min
\end{aligned}
$$

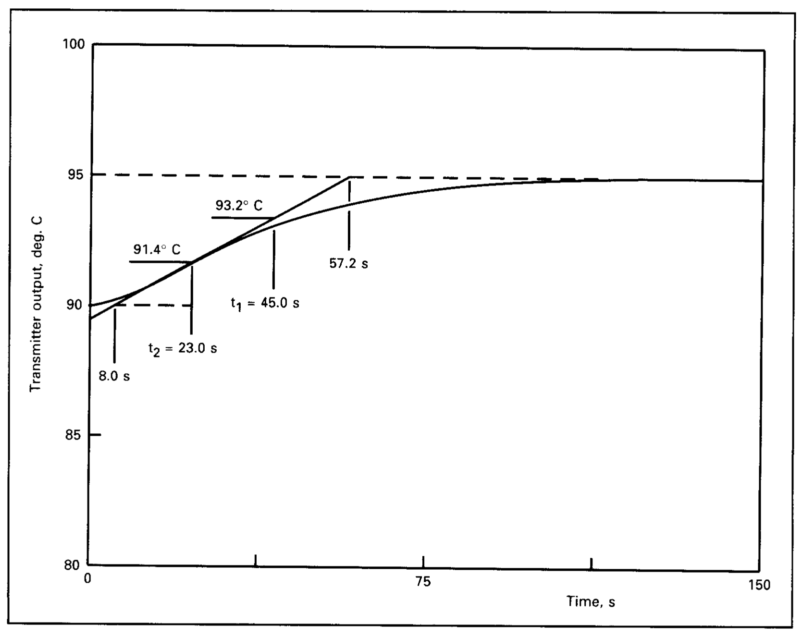


Fig. 3-4. Graphical Determination of Time Constant and Dead Time from Step Response of Steam Heater.

➢ QDR Tuning: Steam Heater

☞ Use process parameters estimated by **tangent** method

$$\text{FOPDT model:} \quad G(s) = \frac{1\ (\%/\%)\ e^{-0.13s}}{0.82s + 1}$$

☞ Controller tuning parameters:

	K_c (%/%)	T_I (min)	T_D (min)
P	6.2	-	-
PI	5.5	0.44	-
PID series	7.4	0.27	0.07

☞ A 10^oC step increase in process inlet temperature:

☞ The **P** and **PI** controllers:

produce about the same maximum initial deviation

☞ The **PID** controller:

➠ Could give smaller initial deviation

➠ Could maintain the temperature closer to the set point for the entire response

➠ Produce about *one third* as much IAE as PI controller

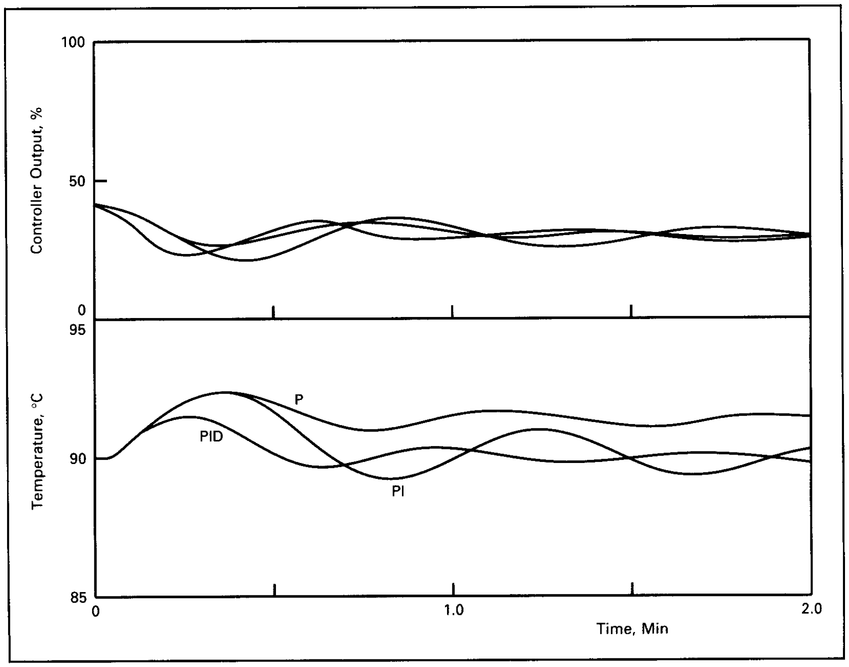


Fig. 4-2. Disturbance Response of P, PI, and PID Controllers Tuned for Quarter-Decay Ratio.

➢ Open-Loop Testing: Heat Exchanger

☞ Process dynamics:

$$G(s) \;=\; \left(\frac{50}{30s+1}\frac{^oC}{\text{kg/s}}\right)\left(\frac{1}{10s+1}\frac{\%\text{TO}}{^oC}\right)\left(\frac{0.016}{3s+1}\frac{\text{kg/s}}{\%\text{CO}}\right)$$

☞ Open-loop test:

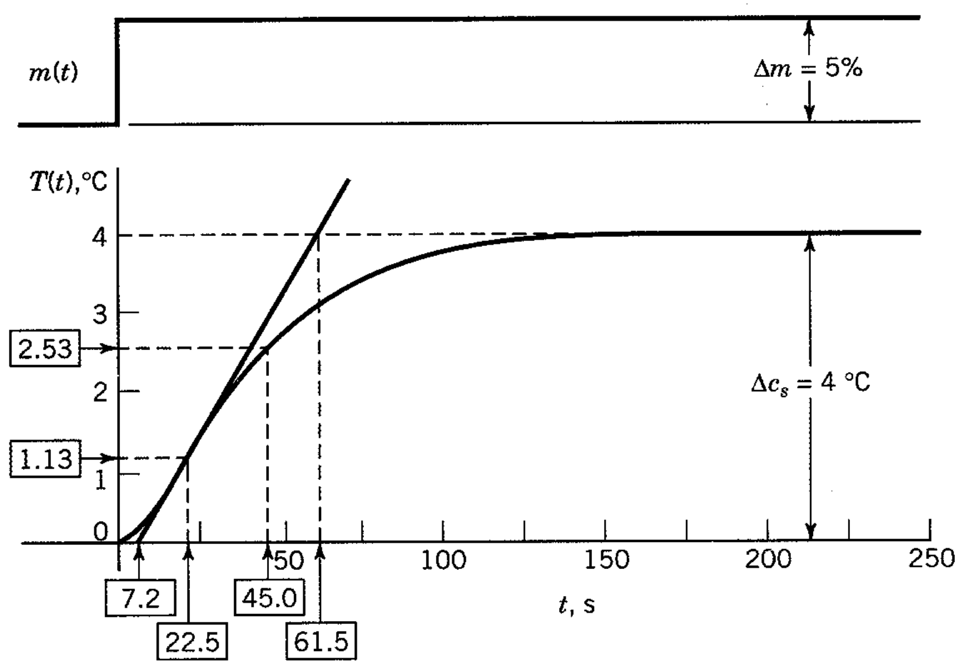


Figure 7-2.7 Step response for the heat exchanger temperature (Example 7-2.1).

☞ **FOPDT models**:

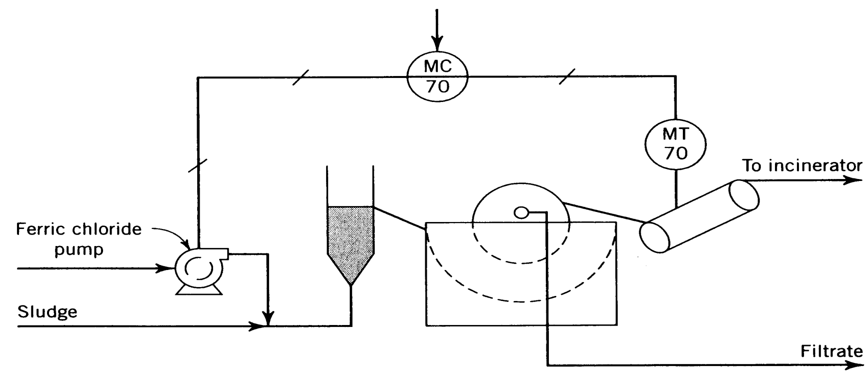
$$\begin{aligned}
K &= \frac{4^oC}{5\%} \times \frac{100\%\text{TO}}{(150-50)^oC} = 0.80\frac{\%\text{TO}}{\%\text{CO}} \\
G_1(s) &= \frac{0.80e^{-7.2s}}{54.3s+1} \quad \text{fit 1} \\
G_2(s) &= \frac{0.80e^{-7.2s}}{37.8s+1} \quad \text{fit 2} \\
G_3(s) &= \frac{0.80e^{-11.2s}}{33.8s+1} \quad \text{fit 3} \\
G_4(s) &= \frac{0.80e^{-12s}}{33.6s+1} \quad \text{fit 4}
\end{aligned}$$

$$\begin{aligned}
d = 7.2 \quad t_3 = 61.5 \quad &\Rightarrow \quad \tau = 61.5 - 7.2 &= 54.3 \\
C(t_2) = 0.632(4) = 2.53 \quad t_2 = 45 \quad &\Rightarrow \quad \tau = 45 - 7.2 &= 37.8 \\
t_1 = 22.5 \quad t_2 = 45 \quad &\Rightarrow \quad \tau = 3(45 - 22.5)/2 &= 33.8 \\
&\quad\quad\; d = 45 - 33.8 &= 11.2 \\
t_{1/3} = 25 \quad t_{2/3} = 49 \quad &\Rightarrow \quad \tau = 1.4(49 - 25)/2 &= 33.6 \\
&\quad\quad\; d = 49 - 1.1\tau &= 12
\end{aligned}$$

FOPDT Model Identification

Example: A Vacuum Filter

Consider a vacuum filter shown below. This process is part of a waste treatment plant. The sludge enters the filter at about 5% solids. In the vacuum filter, the sludge is de-watered to about 25% solids. The filterability of the sludge in the rotating filter depends on the pH of the sludge entering the filter. One way to control the moisture of the sludge to the incinerator is by adding chemicals (ferric chloride) to the sludge feed to maintain the necessary pH. Fi.P7-1 shows a proposed control scheme. The moisture transmitter has a range of 55% to 95%. The following data have been obtained from a step test on the output of the controller (MC70) of $+12.5\%$CO.



Response ($m(t)\%$) to $\Delta u = 12.5\%$

Time (min)	Moisture (%)	Time (min)	Moisture (%)
0.0	75.0	10.5	70.9
1.0	75.0	11.5	70.3
1.5	75.0	13.5	69.3
2.5	75.0	15.5	68.6
3.5	74.9	17.5	68.0
4.5	74.6	19.5	67.6
5.5	74.3	21.5	67.4
6.5	73.6	25.5	67.1
7.5	73.0	29.5	67.0
8.5	72.3	33.5	67.0
9.5	71.6		

When the input moisture to the filter was changed by 2.5%. the following data were obtained.

1. Draw a block diagram for the moisture control loop. Include the possible disturbances.

2. Use fit-3 to estimate parameters of FOPDT models of the two transfer functions.

3. Give an idea of the controllability of the output moisture. What is the correct controller action.

4. Obtain the gain of a proportional controller for minimum minimum IAE response. Calculate the offset for a 5% change in inlet moisture.

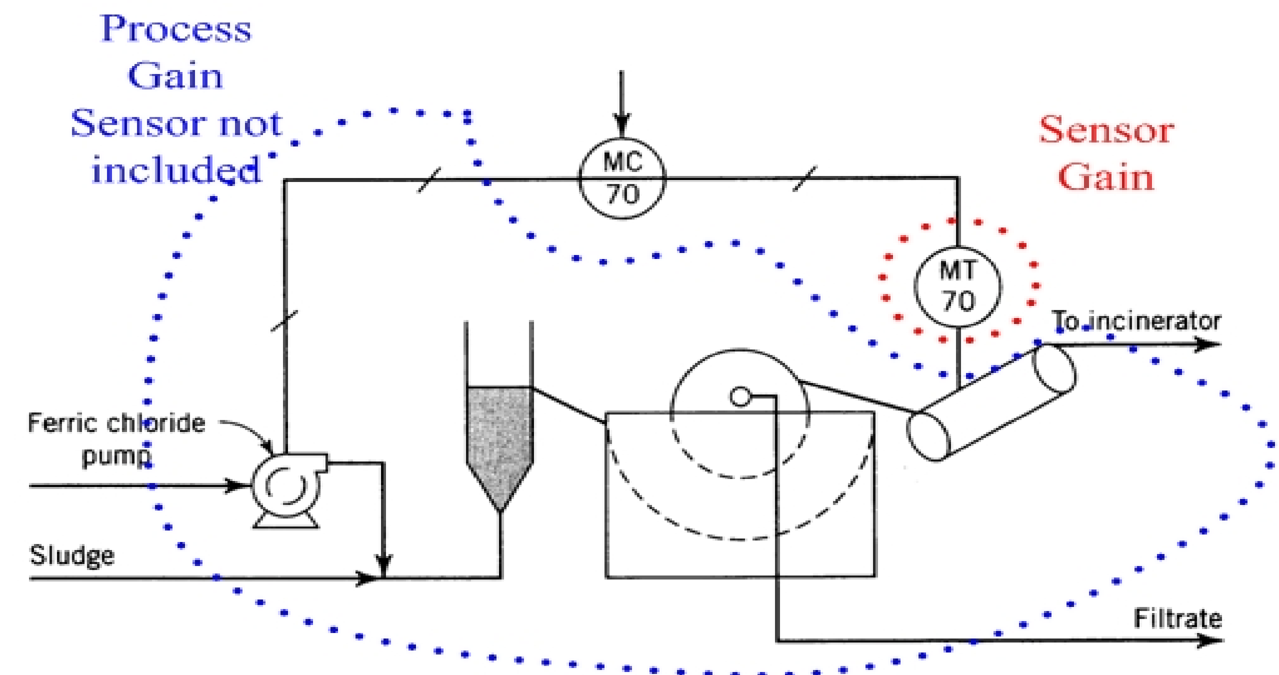
Response $(m(t)\%)$ to $\Delta m_i = 2.5\%$

Time (min)	Moisture (%)	Time (min)	Moisture (%)
0.0	75.0	11	75.9
1.0	75.0	12	76.1
2.0	75.0	13	76.2
3.0	75.0	14	76.3
4.0	75.0	15	76.4
5.0	75.0	17	76.6
6.0	75.1	19	76.7
7.0	75.3	21	76.8
8.0	75.4	25	76.9
9.0	75.6	29	77.0
10.	75.7	33	77.0

FOPDT Model Identification

A Vacuum Filter (solution)

$$
\begin{aligned}
K_T &= \frac{100-0}{95-55} = 2.5 \ \frac{\%\text{ TO}}{\%\text{ moist}} \\
\Delta m &= 12.5\ \%\text{CO} \qquad \Delta x = 67.0 - 75.0 = -8.0\ \%\text{ moist} \\
K_{\text{pump}} K_P &= \frac{-8.0}{12.5} = -0.64\ \frac{\%\text{ moist}}{\%\text{ CO}} \\
K_1 &= K_{\text{pump}} K_P K_T = -1.60 \frac{\%\text{ TO}}{\%\text{ CO}}
\end{aligned}
$$



G

$0.5s + 1$

FOPDT Model Identification

A Vacuum Filter (solution)

$$
\begin{aligned}
K_T &= \frac{100-0}{95-55} = 2.5 \ \frac{\%\ \text{TO}}{\%\ \text{moist}} \\
\Delta m &= 12.5\ \%\text{CO} \qquad \Delta x = 67.0 - 75.0 = -8.0\ \%\ \text{moist} \\
K_{\text{pump}} K_P &= \frac{-8.0}{12.5} = -0.64\ \frac{\%\ \text{moist}}{\%\ \text{CO}} \\
K_1 &= K_{\text{pump}} K_P K_T = -1.60 \frac{\%\ \text{TO}}{\%\ \text{CO}} \\
\Delta x_1 &= 75.0 + 0.283(-8.0) = 72.73\ \%\ \text{moist} \\
t_1 &= 7.5 + \frac{72.73 - 73.0}{72.3 - 73.0}(1.0) = 7.88\ \text{min} \\
\Delta x_2 &= 75.0 + 0.632(-8.0) = 69.94\ \%\ \text{moist} \\
t_2 &= 7.5 + \frac{69.94 - 73.0}{69.3 - 70.3}(2.0) = 12.21\ \text{min} \\
\tau &= \frac{3}{2}(12.21 - 7.88) = 6.5\ \text{min} \\
\theta &= 12.21 - 6.5 = 5.7\ \text{min} \\
G_p(s) &= \frac{-1.6e^{-5.7s}}{6.5s + 1}
\end{aligned}
$$

FOPDT Model Identification

A Vacuum Filter (solution)

$$
\begin{aligned}
\Delta x_i &= 2.5\ \%\%\ \text{moist} \qquad \Delta x = 77.0 - 75.0 = 2.0\ \%\ \text{moist} \\
K_{P_2} &= \frac{2.0}{2.5} = 0.8\ \frac{\%\ \text{moist}}{\%\ \text{moist}} \\
K_2 &= K_{P_2} K_T = 2.0 \frac{\%\ \text{TO}}{\%\ \text{moist}} \\
\Delta x_1 &= 75.0 + 0.283(2.0) = 75.57\ \%\ \text{moist} \\
t_1 &= 8 + \frac{75.57 - 75.4}{75.6 - 75.4}(1.0) = 8.8\ \text{min} \\
\Delta x_2 &= 75.0 + 0.632(2.0) = 76.26\ \%\ \text{moist} \\
t_2 &= 13 + \frac{76.26 - 76.2}{76.3 - 76.2}(1.0) = 13.6\ \text{min} \\
\tau &= \frac{3}{2}(13.6 - 8.8) = 7.2\ \text{min} \\
\theta &= 13.6 - 7.2 = 6.4\ \text{min} \\
G_\ell(s) &= \frac{2.0e^{-6.4s}}{7.2s + 1}
\end{aligned}
$$

FOPDT Model Identification

A Vacuum Filter (solution)

$$\frac{\theta}{\tau} = \frac{5.7}{6.5} = 0.88 \quad \text{(quite high ratio, not very controllable)}$$

$$K_c = \frac{0.902}{-1.60}\left(\frac{5.7}{6.5}\right)^{-0.985} = -0.64 \ \frac{\%\ \text{CO}}{\%\ \text{TO}}$$

$$\text{offset} = 0 - \frac{2.0}{1+(-0.64)(-1.60)}(5) = -4.9 \ \%\ \text{TO} \ (-2.0\ \%\ \text{moist})$$

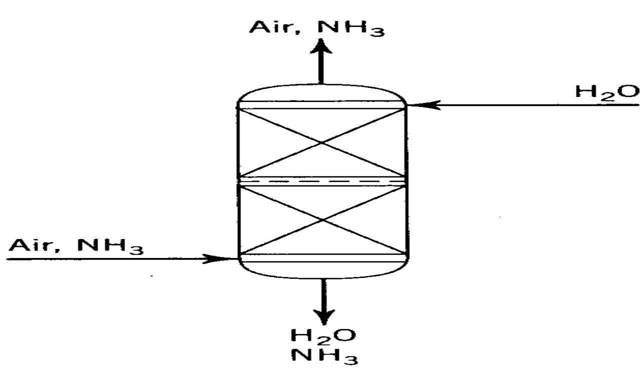
$$K_c = \frac{0.902}{-1.60}\left(\frac{5.7}{6.5}\right)^{-1} = -0.64 \ \frac{\%\ \text{CO}}{\%\ \text{TO}}$$

$$\tau = 3.33(5.7) = 19 \ \text{min}$$

FOPDT Model Identification

Example: An Absorber

A gas with a composition of 90mole% air and 10mole% ammonia is entering an absorber. Before this gas is vented to the atmosphere, it is necessary to remove most of the ammonia from it. This will be done by absorbing it with water. The absorber has been designed so that the outlet ammonia in the vapor is 50 ppm. The following table gives the response to a step change in water flow to the absorber. Approximate the response of the absorber with a first-order-plus-dead-time model.



Time (s)	Water Flow, gpm	Outlet Conc
0	250	50.00
0	200	50.00
20	200	50.00
30	200	50.12
40	200	50.30
50	200	50.60
60	200	50.77
70	200	50.90
80	200	51.05
90	200	51.20
100	200	51.26
110	200	51.35
120	200	51.48
130	200	51.55
140	200	51.63
160	200	51.76
180	200	51.77
250	200	51.77

FOPDT Model Identification

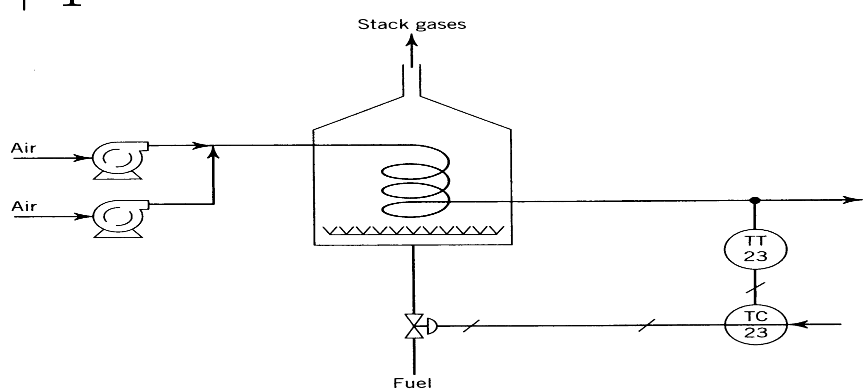
Example: An Absorber (solution)

$$
\begin{aligned}
K_T &= \frac{100-0}{200-0} = 0.5\ \frac{\%\ \text{TO}}{\text{ppm}} \qquad K_V = -\frac{500-0}{100-0} = -5\ \frac{\text{gpm}}{\%\ \text{CO}} \\
\Delta f &= 200-250 = -50\ \text{gpm} \qquad \Delta y = 51.77-50.00 = 1.77\ \text{ppm} \\
K_P &= \frac{\Delta y}{\Delta f} = \frac{1.77}{-50} = -0.0345\ \frac{\text{ppm}}{\text{gpm}} \\
K &= K_V K_P K_T = \left(-5\ \frac{\text{gpm}}{\%\ \text{CO}}\right)\left(-0.0345\ \frac{\text{ppm}}{\text{gpm}}\right)\left(0.5\ \frac{\%\ \text{TO}}{\text{ppm}}\right) = 0.0885\ \frac{\%\ \text{TO}}{\%\ \text{CO}} \\
\Delta y_1 &= 50.0 + 0.283(1.77) = 50.50\ \text{ppm} \\
t_1 &= 40.0 + \frac{50.50-50.30}{50.60-50.30}(10) = 46.70\ \text{sec} \\
\Delta y_2 &= 50.0 + 0.632(1.77) = 51.13\ \text{ppm} \\
t_2 &= 80.0 + \frac{51.1351.05}{51.20-51.05}(10) = 85.28\ \text{sec} \\
\tau &= \frac{3}{2}(85.28-46.70) = 57.9\ \text{sec} = 0.96\ \text{min} \\
\theta &= 85.28-57.9 = 27.4\ \text{sec} = 0.46\ \text{min} \qquad G_p(s) = \frac{-0.0354e^{-0.46s}}{0.96s+1}
\end{aligned}
$$

FOPDT Model Identification

Example: A Furnace

Consider the furnace, which is used to heat the supply air to a catalyst regenerator. The temperature transmitter is calibrated for 300^oF to 500^oF. The following response data were obtained for a step change of $+5\%$ in the output of the controller. Fit the process data by a first-order-plus-dead-time model, $\dfrac{K_p e^{-\theta s}}{\tau s + 1}$.



time (min)	$T(t)^oF$	time (min)	$T(t)^oF$
0	425	5.5	436.6
0.5	425	6.0	437.6
1.0	425	7.0	439.4
2.0	425	8.0	440.7
2.5	426.4	9.0	441.7
3.0	428.5	10.0	442.5
3.5	430.6	11.0	443.0
4.0	432.4	12.0	443.5
4.5	434	14.0	444.1
5.0	435.3	20.0	445.0

FOPDT Model Identification

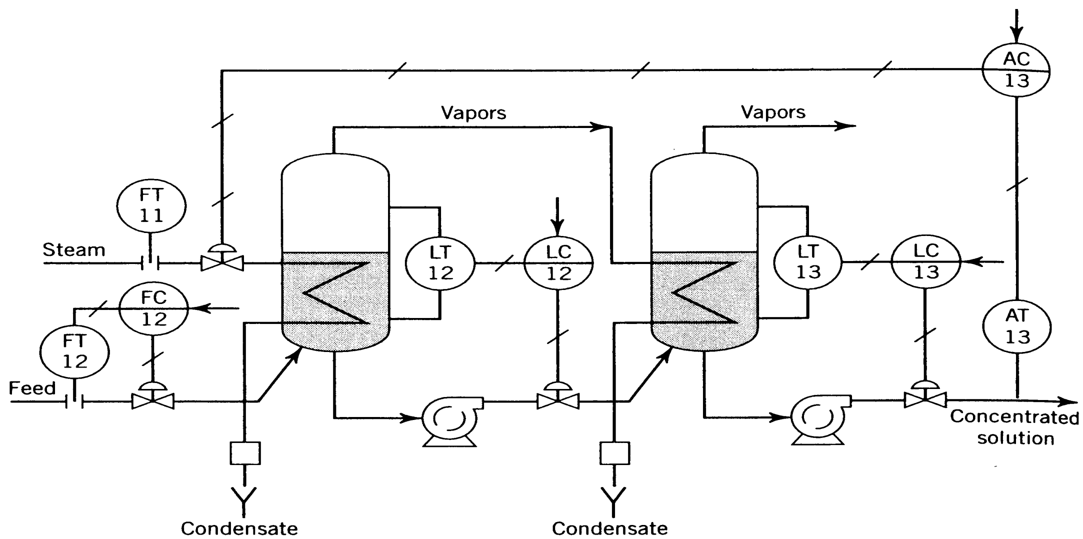
Example: A Furnace (solution)

$$
\begin{aligned}
K_T &= \frac{100-0}{500-300} = 0.5 \ \frac{\%\ \text{TO}}{^o\text{F}} \\
\Delta m &= 5\ \%\text{CO} \qquad \Delta T = 445-425 = 20\ ^o\text{F} \\
K_{P_1} &= \frac{20}{5} = 4.0\ \frac{^o\text{F}}{\%\ \text{CO}} \\
K_1 &= K_{P_1}K_T = 4.0(0.50) = 2.0\ \frac{\%\ \text{TO}}{\%\ \text{CO}} \\
\Delta T_1 &= 425 + 0.283(20) = 430.7\ ^o\text{F} \\
t_1 &= 3.5 + \frac{430.7-430.6}{432.4-430.6}(0.5) = 3.52\ \text{min} \\
\Delta T_2 &= 425 + 0.632(20) = 437.6\ ^o\text{F} \\
t_2 &= 6.0\ \text{min} \\
\tau &= \frac{3}{2}(6.0-3.52) = 3.72\ \text{min} \\
\theta &= 6.0 - 3.72 = 2.28\ \text{min} \\
G_p(s) &= \frac{2.0e^{-2.28s}}{3.72s+1}
\end{aligned}
$$

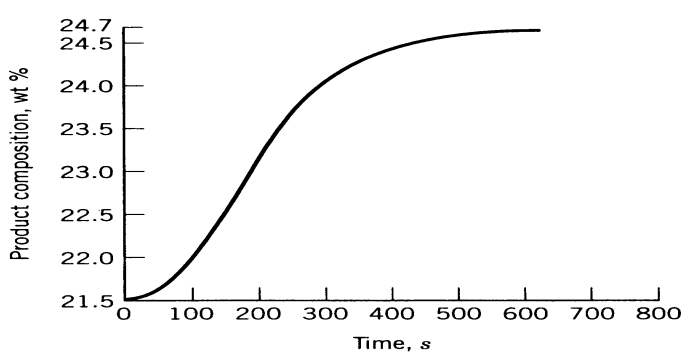
FOPDT Model Identification

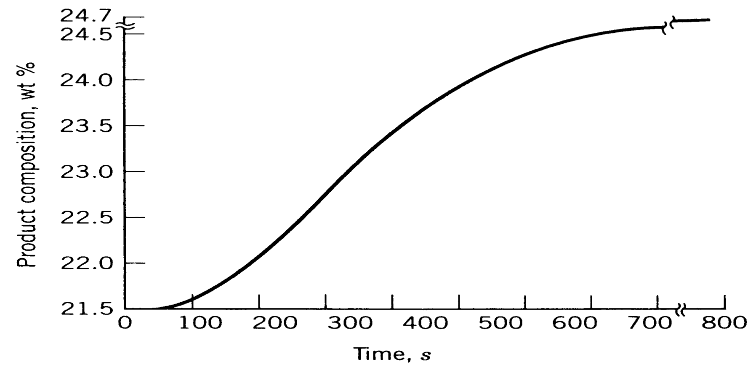
Example: A Double-Effect Evaporator

Consider the typical control system for the double-effect evaporator shown below. The composition of the product out of the last effect is controlled by manipulating the steam to the first effect. The design feed rate and composition are $50,000$ lb/hr and 5.0 weight percent. The composition sensor/transmitter has a range of $10 \sim 35$ weight percent.



The following two figures show the open-loop response of product composition for a change of 2.5% in controller output (left) and a change of 0.75% by weight in composition of solution entering first effect (right), respectively.





1. What should be the fail-safe position of the control valve ? What is the correct controller action ?

2. Draw a complete block diagram with the transfer of each block (use fit-1).

3. Tune a series PID controller for quarter decay ratio response.

4. Tune a PI controller for 5% overshoot, using the controller synthesis method.

FOPDT Model Identification

Example: A Double-Effect Evaporator (soln)

$$
\begin{aligned}
K_T &= \frac{100-0}{35-10} = 4.0 \ \frac{\%\ \text{TO}}{\text{wt}\ \%} \\
\Delta m &= 2.5\ \%\text{CO} \qquad \Delta x = 24.7-21.5 = 3.2\ \text{wt}\ \% \\
K_V K_{P_1} &= \frac{3.2}{2.5} = 1.28\ \frac{\text{wt}\ \%}{\%\ \text{CO}} \\
K_1 &= K_V K_{P_1} K_T = 1.28(4.0) = 5.12\ \frac{\%\ \text{TO}}{\%\ \text{CO}} \\
\Delta x_1 &= 21.5+0.283(3.2) = 22.4\ \text{wt}\ \% \\
t_1 &= 250\ \text{sec} \\
\Delta x_2 &= 21.5+0.632(3.2) = 23.5\ \text{wt}\ \% \\
t_2 &= 415\ \text{sec} \\
\tau &= \frac{3}{2}(415-250) = 248\ \text{sec} = 4.12\ \text{min} \\
\theta &= 415-248 = 167\ \text{sec} = 2.79\ \text{min} \\
G_p(s) &= \frac{5.12e^{-2.79s}}{4.12s+1}
\end{aligned}
$$

$$\begin{aligned}
\Delta x_F &= 0.75\ \%\text{wt}\ \% \qquad \Delta x = 24.7 - 21.5 = 3.2\ \text{wt}\ \% \\
K_V K_{P_1} &= \frac{3.2}{0.75} = 4.27\ \frac{\text{wt}\ \%}{\%\ \text{CO}} \\
K_2 &= K_{P_2} K_T = 4.27(4.0) = 17.07\ \frac{\%\ \text{TO}}{\text{wt}\ \%} \\
\Delta x_1 &= 22.4\ \text{wt}\ \% \qquad t_1 = 140\ \text{sec} \\
\Delta x_2 &= 23.5\ \text{wt}\ \% \qquad t_2 = 230\ \text{sec} \\
\tau &= \frac{3}{2}(230 - 140) = 135\ \text{sec} = 2.25\ \text{min} \\
\theta &= 230 - 135 = 95\ \text{sec} = 1.58\ \text{min} \\
G_\ell(s) &= \frac{17.07 e^{-1.58s}}{2.25s + 1}
\end{aligned}$$

QDR Tuning: A Summary

QDR Tuning Based on Ultimate Gain and Period

Controller	Gain	Integral Time	Derivative Time
P	$K_c = 0.50K_{cu}$	-	-
PI	$K_c = 0.45K_{cu}$	$T_I = T_U/1.2$	-
PID parallel	$K_c = 0.75K_{cu}$	$T_I = T_U/1.6$	$T_D = T_U/10$
PID series	$K_c^{'} = 0.60K_{cu}$	$T_I^{'} = T_U/2.0$	$T_D^{'} = T_U/8$

QDR Tuning: A Summary

QDR Tuning Based on 2-Parameters Model

	gain	integral time	derivative time
P	$K_c = \frac{1}{a}$	-	-
PI	$K_c = 0.9\ \frac{1}{a}$	$T_I = 3.33\ d$	-
PID series	$K_c' = 1.2\ \frac{1}{a}$	$T_I' = 2.0\ \ d$	$T_D' = 0.5\ d$

QDR Tuning: A Summary

QDR Tuning Based on FOPDT Model

	gain	integral	derivative
P	$K_c = \frac{\tau}{K\ d}$	-	-
PI	$K_c = 0.9\ \frac{\tau}{K\ d}$	$T_I = 3.33\ d$	-
PID series	$K_c^{'} = 1.2\ \frac{\tau}{K\ d}$	$T_I^{'} = 2.0\ \ d$	$T_D^{'} = 0.5\ d$

QDR Tuning: A Summary

Major Conclusions

➢ $K_c \propto 1/K$

- ☞ Loop response depends on **loop gain** $(K_c \cdot K)$
- ☞ Gain of any element is changed because of recablication, re-sizing, or nonlinearity
  $\Rightarrow$ response would change unless K_c is readjusted

➢ K_c must be reduced when $R \equiv d/\tau$ increases

- ☞ Controllability of the loop decreases when $R \uparrow$
- ☞ Uncontrollability parameter of the loop: $R = d/\tau$
- ☞ A long dead time means loop is less controllable
  only if time constant is short
- ☞ A loop with a dead time of several minutes would be just as controllable as one with a dead time of a few seconds if the **uncontrollability parameter** for both loops is the same

- Speed of the controller (determined by T_I and T_D) must match speed of process response (here, d)

- If performance of a well-tuned controller was to deteriorate under operation $\Rightarrow$ check a change in:
  - Process gain
  - Uncontrollability parameter
  - Speed of response

QDR Tuning: A Summary

Applying the QDR Tuning Formula

- Range of process uncontrollability parameter: $0.1 < R < 0.3$
- Apply to **series PID** controller
- Formula were developed for continuous analog controllers
- For digital controller: $d \ \Rightarrow \ d + \dfrac{T}{2}$ (T: sampling period)
- Increasing T would reduce controllability of the loop:

  $$R = \frac{d + \frac{T}{2}}{\tau}$$
- For most loops, control performance does not improve much when sampling time is reduced beyond one tenth of time constant

Tuning for Min Error Integrals

Tuning for Minimum Error Integrals

➢ Major Limitations of QDR Tuning

- ☞ Narrow range of usable uncontrollability parameter $(0.1 \sim 0.3)$
- ☞ An infinite no. of combinations of PI, PID tuning parameters

The Minimum Error Integrals

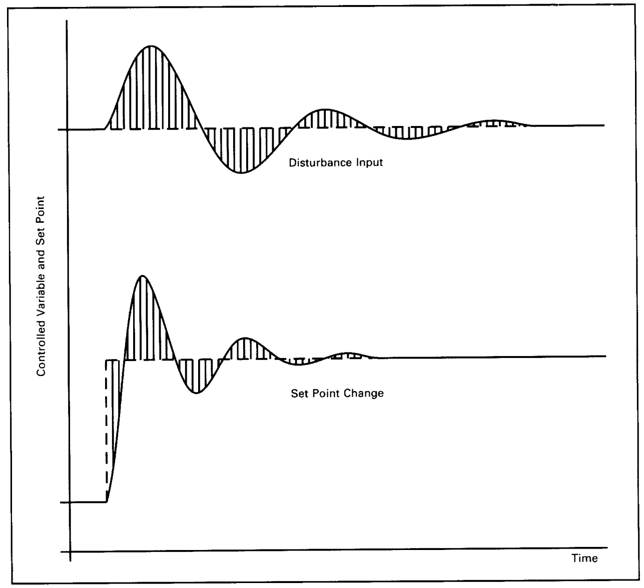


Fig. 4-1. Definition of Error Integral Criteria.

➢ Integral of Absolute value of the Error (**IAE**)

$$\textbf{IAE} = \int |e(t)| dt$$

➢ Integral of Squared Error (**ISE**)

$$\textbf{ISE} = \int e^2(t) dt$$

➢ Integral of Time-weighted Absolute value of the Error (**ITAE**)

$$\textbf{ITAE} = \int t|e(t)|dt$$

➢ Integral of Time-weighted Square of the Error (**ITSE**)

$$\textbf{ITSE} = \int te^2(t)dt$$

➢ Aspects of closed-loop response for different MEIs:

☞ **ISE, ITSE**: weight large errors more than IAE, ITAE
$\Rightarrow$ tighter, more oscillatory response

☞ **ITAE, ITSE**: put more weight on errors at end of response
$\Rightarrow$ having larger initial deviations than **IAE, ISE**

Tuning for Disturbance Changes

➢ Tuning for **min. IAE** on **disturbance** inputs (Lopez et al., 1967)

	Gain	Integral Time	Derivative Time
P	$KK_c = 0.902\left(\frac{d}{\tau}\right)^{-0.985}$	-	-
PI	$KK_c = 0.984\left(\frac{d}{\tau}\right)^{-0.985}$	$\frac{T_I}{\tau} = 1.645\left(\frac{d}{\tau}\right)^{0.707}$	-
PID parallel	$KK_c = 1.435\left(\frac{d}{\tau}\right)^{-0.921}$	$\frac{T_I}{\tau} = 1.139\left(\frac{d}{\tau}\right)^{0.749}$	$\frac{T_D}{\tau} = 0.482\tau\left(\frac{d}{\tau}\right)^{1.137}$

➢ Tuning for **min. ITAE** on **disturbance** inputs

	Gain	Integral Time	Derivative Time
P	$KK_c = 0.490\left(\frac{d}{\tau}\right)^{-1.084}$	-	-
PI	$KK_c = 0.859\left(\frac{d}{\tau}\right)^{-0.977}$	$\frac{T_I}{\tau} = 1.484\left(\frac{d}{\tau}\right)^{0.680}$	-
PID parallel	$KK_c = 1.357\left(\frac{d}{\tau}\right)^{-0.947}$	$\frac{T_I}{\tau} = 1.188\left(\frac{d}{\tau}\right)^{0.738}$	$\frac{T_D}{\tau} = 0.381\left(\frac{d}{\tau}\right)^{0.995}$

➢ Tuning for **min. ISE** on **disturbance** inputs

	Gain	Integral Time	Derivative Time
P	$KK_c = 1.411 \left(\frac{d}{\tau}\right)^{-0.917}$	-	-
PI	$KK_c = 1.305 \left(\frac{d}{\tau}\right)^{-0.959}$	$\frac{T_I}{\tau} = 2.033 \left(\frac{d}{\tau}\right)^{0.739}$	-
PID parallel	$KK_c = 1.495 \left(\frac{d}{\tau}\right)^{-0.945}$	$\frac{T_I}{\tau} = 0.908 \left(\frac{d}{\tau}\right)^{0.771}$	$\frac{T_D}{\tau} = 0.560 \left(\frac{d}{\tau}\right)^{1.006}$

➢ Tuning for **min. ITSE** on **disturbance** inputs $(None)$

Conclusions for MEI Tunings for Disturbance

- **ISE** formula result in the tightest tuning (highest gain, shortest integral time)
- **ITAE** results in the loosest tuning
- **IAE** results in intermediate tuning
- **ITSE** would probably fall between **IAE** and ISE in tightness of tuning

- K_c: inversely proportional to process gain
- Optimum loop gain decreases with its uncontrollability parameter
- Speed of response of controller must match speed of response of process
- **MEI** formula relate T_I, T_D to τ rather than d (unlike **QDR**)

Tuning for Setpoint Changes (Rovira 1981)

➢ Setpoint suddenly changed
$\Rightarrow$ error changes from zero to a finite value instantaneously

➢ Disturbances input
$\Rightarrow$ error grows gradually $\Rightarrow$ higher controller gains

➢ **Tuning for disturbance** assume same rate of response for disturbance input and for change in controller output

➢ **Tuning for setpoint**:
smaller gains, smaller derivative times, longer integral times

➢ Tuning for **min. IAE** on **setpoint** inputs

	Gain	Integral Time	Derivative Time
PI	$KK_c = 0.758\left(\frac{d}{\tau}\right)^{-0.861}$	$\frac{T_I}{\tau} = \frac{1}{1.02 - 0.323\left(\frac{d}{\tau}\right)}$	-
PID parallel	$KK_c = 1.086\left(\frac{d}{\tau}\right)^{-0.869}$	$\frac{T_I}{\tau} = \frac{1}{0.74 - 0.130\left(\frac{d}{\tau}\right)}$	$\frac{T_D}{\tau} = 0.348\left(\frac{d}{\tau}\right)^{0.914}$

➢ Tuning for **min. ITAE** on **setpoint** inputs

	Gain	Integral Time	Derivative Time
PI	$KK_c = 0.586\left(\frac{d}{\tau}\right)^{-0.916}$	$\frac{T_I}{\tau} = \frac{1}{1.03 - 0.165\left(\frac{d}{\tau}\right)}$	-
PID parallel	$KK_c = 0.965\left(\frac{d}{\tau}\right)^{-0.855}$	$\frac{T_I}{\tau} = \frac{1}{0.80 - 0.147\left(\frac{d}{\tau}\right)}$	$\frac{T_D}{\tau} = 0.308\left(\frac{d}{\tau}\right)^{0.929}$

Application of Minimum Error Integral Formula

- Range of process uncontrollability parameter: $0.1 \sim 1.0$
- Apply to the **parallel PID** controllers
- Tuning formula developed for continuous analog controllers
- When applied to digital controllers: $d \Rightarrow d + \frac{T}{2}$
- Many digital controllers offer option of having either *the D mode or the P mode or both* act on **CV** instead of on **error**
- P and D mode act on CV $\Rightarrow$ use formula for disturbance (tighter tuning without danger of excessive overshoot)
- Slave controllers in cascade: use setpoint tuning
  P mode act on error
  $\Rightarrow$ CO responds quickly to setpoint changes

Comparative Examples of MEI Tunings

➢ Process: Steam Heater

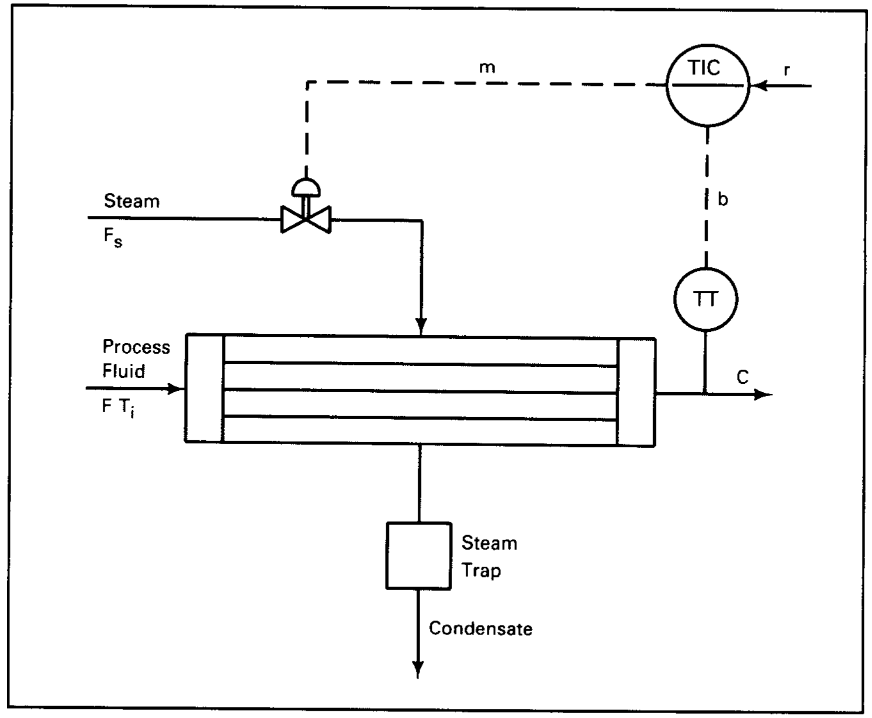


Fig. 3-1. Sketch of Temperature Control of Steam Heater.

➢ **Model parameters**: (Gain = 1.0 %TO/%CO)

	Tangent Method	Tangent and Point Method	Two-Point Method
Time Constant, min	0.82	0.62	0.55
Dead Time, min	0.13	0.13	0.20
Uncontrollability	0.16	0.21	0.36

➢ **Ziegler and Nichols** used **tangent** method to develop their empirical formulas, working with actual processes
$\Rightarrow$ use **tangent** method for **QDR** response

➢ **Lopez/Rovira** developed **MEI** formulas using true FOPDT models
$\Rightarrow$ any method can be used for determining a **FOPDT**

➢ **Tangent** method will result in tightest tuning parameters

➢ **Two-point** method will result in most conservative tuning

MEI PI Responses for Disturbance Inputs

➢ Use process parameters estimated by *tangent-and-point method*

➢ PI tuning parameters:

minimum IAE	4.4	0.34
minimum ISE	5.7	0.40
minimum ITAE	3.8	0.32

➢ Response for a 10^oC step increase in T_i:

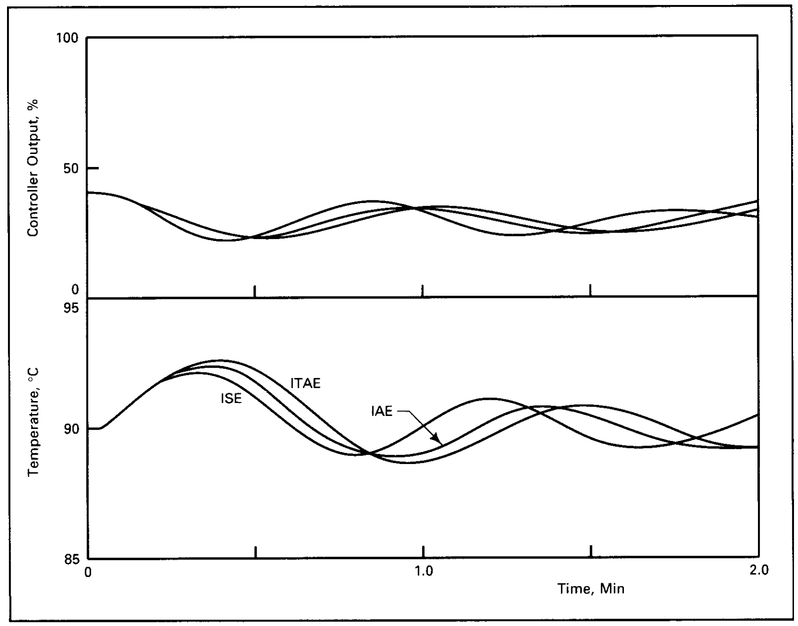


Fig. 4-3. Disturbance Response for PI Controller Tuned for Minimum Error Integral Criteria: IAE, ISE, and ITAE.

➢ **NO** practical difference between the three responses

➢ **ISE** tuning produces a slightly faster response

➢ **IAE** tuning and response are intermediate of the other two

➢ Overall differences between the three sets of tuning parameters are *insignificant*

Disturbance versus Setpoint Tuning

➢ Compare performance of a **series PID** controller tuned by formula for **min IAE** on **disturbance and set point** inputs

➢ Steam heater parameters estimated by **tangent-and-point method**

➢ Series PID tuning parameters:

☞ parallel PID controller, disturbance tuning:

$$K_c = 5.9\ \%/\% \quad T_I = 0.22\ min \quad T_D = 0.05\ min$$

☞ parallel PID controller, set point tuning:

$$K_c = 4.1\ \%/\% \quad T_I = 0.87\ min \quad T_D = 0.05\ min$$

☞ the equivalent series PID controller parameters:

	$K_c^{'}$ (%/%)	$T_I^{'}$ (min)	$T_D^{'}$ (min)
Disturbance Tuning	3.8	0.14	0.08
Set Point Tuning	3.8	0.81	0.06

➢ $T_{I_{\text{set point}}}^{'} \gg T_{I_{\text{disturbance}}}^{'}$

Disturbance versus Setpoint Tuning

➢ **Step disturbance response** (10^oC $\uparrow$ in T_i)

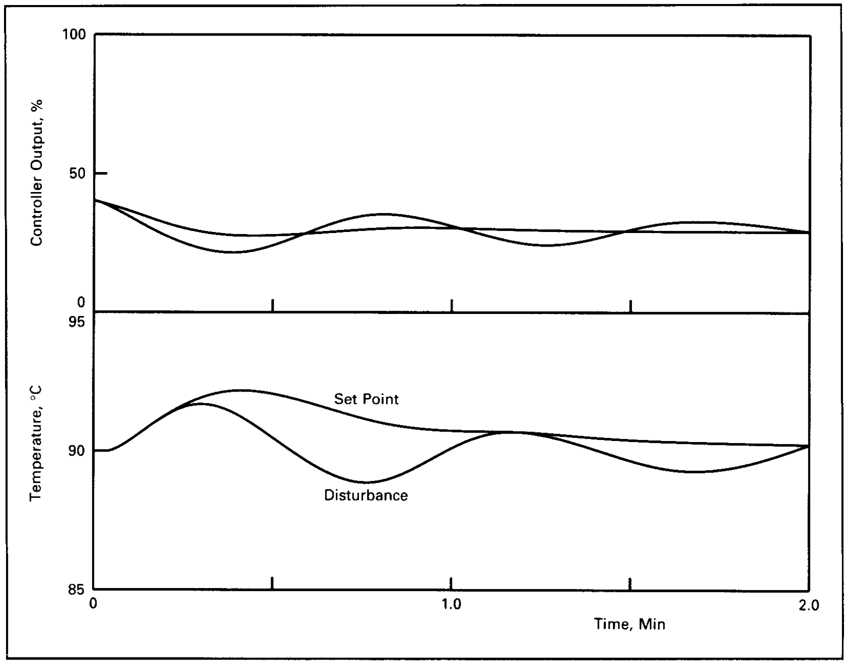


Fig. 4-4. Disturbance Response of PID Controller Tuned for Minimum IAE on Disturbance and on Set Point Inputs.

➢ **Disturbance tuning** results in:

- ☞ A smaller maximum deviation
- ☞ Quicker return to set point
- ☞ More oscillatory response
- ☞ IAE: 64% of the IAE for SP tuning

➢ **Step set point response** (5^oC $\uparrow$ in S.P.)

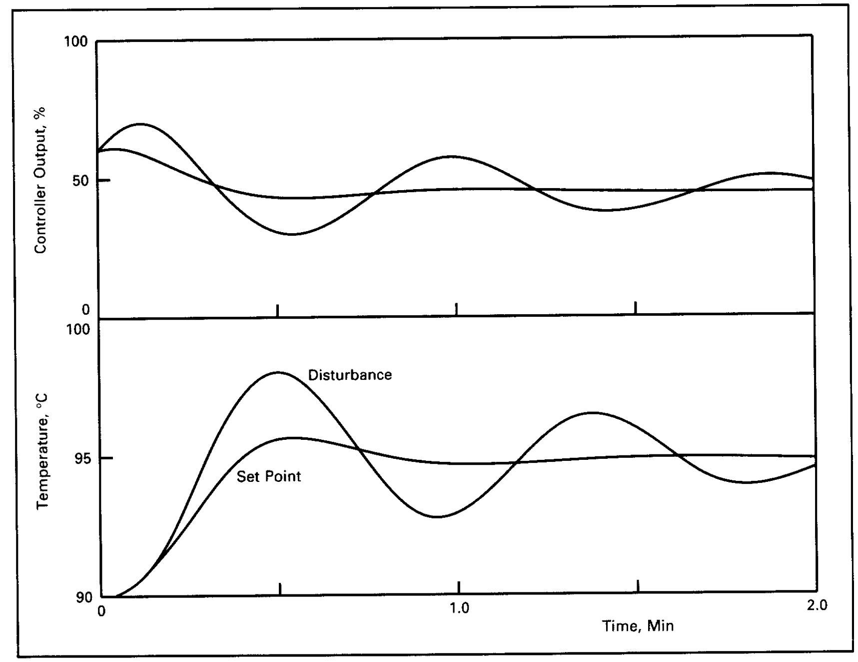


Fig. 4-5. Set Point Response of PID Controller Tuned for Minimum IAE on Disturbance and on Set Point.

➢ **SP tuning** results in:

- ☞ Very little overshoot
- ☞ Better approach to new SP
- ☞ IAE: 48% of the IAE for disturbance tuning

Disturbance versus Setpoint Tuning

Discussion

- ➢ Each set of tuning formula performs better than the other on the input for which it is intended
- ➢ Uncontrollability: 0.22
  $\Rightarrow$ derivative mode results in superior response
- ➢ QDR and various M.E.I. formula for disturbance inputs result in similar tuning parameters

Controller Synthesis

Controller Synthesis

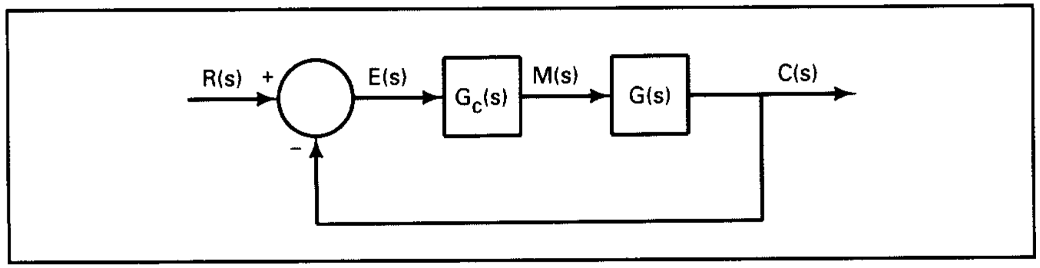


Fig. 5-3. Block Diagram of the General Feedback Control Loop.

➢ $G(s)$: process; G_c: controller; $G_{c\ell}$: closed-loop TF

Given $G(s)$, specific $G_{c\ell(s)} \Rightarrow G_c(s)$

$$\begin{aligned} G_{c\ell}(s) &= \frac{G(s)G_c(s)}{1+G(s)G_c(s)} \\ \Rightarrow \quad G_c(s) &= \frac{1}{G(s)}\,\frac{G_{c\ell}(s)}{1-G_{c\ell}(s)} \end{aligned}$$

➢ Some specific $G(s)$, $G_{c\ell}(s)$ will result in **PID**s

Fast Processes

$$
\begin{aligned}
\text{Process:} \quad G(s) &= K \\
\text{Spec:} \quad G_{c\ell}(s) &= \frac{1}{\tau_c s + 1} \\
\text{Controller:} \quad G_c(s) &= \frac{1}{K\tau_c}\frac{1}{s} \\
&= \frac{K_I}{s} \\
\text{Tuning:} \quad K_I &= \frac{1}{K\tau_c}
\end{aligned}
$$

➢ Fast enough to require only pure **I**ntegral control

➢ **Examples:**

☞ Exit temperature control in reformer furnaces

☞ Some flow control loops

First-Order Processes

$$\begin{aligned}
\text{Process:} \quad G(s) &= \frac{K}{\tau s + 1} \\
\text{Spec:} \quad G_{c\ell}(s) &= \frac{1}{\tau_c s + 1} \\
\text{Controller:} \quad G_c(s) &= \frac{\tau}{K\tau_c}\left[1 + \frac{1}{\tau s}\right] \\
&= K_c\left[1 + \frac{1}{T_i s}\right] \\
\text{Tuning:} \quad K_c &= \frac{\tau}{K\tau_c} \quad T_i = \tau
\end{aligned}$$

➢ Zero offset spec $\longrightarrow$ **I**ntegral mode

➢ **P** mode is added to compensate for process lag

Second-Order Processes

$$
\begin{aligned}
\text{Process:} \quad G(s) &= \frac{K}{(\tau_1 s+1)(\tau_2 s+1)} \qquad (\tau_1 > \tau_2)\\
\text{Spec:} \quad G_{c\ell}(s) &= \frac{1}{\tau_c s+1}\\
\text{Controller:} \quad G_c(s) &= \frac{\tau_1}{K\tau_c}\left[1+\frac{1}{\tau_1 s}\right][\tau_2 s+1]\\
&= K_c\left[1+\frac{1}{T_i s}\right][T_d s+1]\\
\text{Tuning:} \quad K_c &= \frac{\tau}{K\tau_c} \quad T_i=\tau_1 \quad T_d=\tau_2
\end{aligned}
$$

- **D**erivative mode is added to compensate for second lag

  (temperature loops: sensor lag $\rightarrow$ **PID**)

- Integral, Derivative times = two time constants

- **P** mode: adjustable to obtain desired tightness of response

- Lag in derivative unit is not obtained (realizable ?)

Integrating Process

$$
\begin{aligned}
\text{Process:} \quad G(s) &= \frac{K}{s(\tau s+1)} \\
\text{Spec:} \quad G_{c\ell}(s) &= \frac{1}{\tau_c s+1} \\
\text{Controller:} \quad G_c(s) &= \frac{1}{K\tau_c}\left[\tau s+1\right] \\
&= K_c\left[T_d s+1\right] \\
\text{Tuning:} \quad K_c &= \frac{1}{K\tau_c} \quad T_d = \tau
\end{aligned}
$$

- ➢ The required lag on derivative unit is not presented but must be included in actual implementation

- ➢ **P** mode for integrating process:
  - ☞ No offset for set point changes
  - ☞ Disturbances cause offset because no I mode in controller

- ➢ **Example:** liquid level control
  use **P** mode, add **D** mode for lag in sensor, process, valve

Processes with Inverse/Overshoot Response

$$\text{Process:} \quad G(s) = \frac{K(1-\tau_3 s)}{(\tau_1 s+1)(\tau_2 s+1)} \quad (\tau_1 > \tau_2)$$

$$\text{Spec:} \quad G_{c\ell}(s) = \frac{1-\tau_3 s}{\tau_c s+1}$$

$$\text{Controller:} \quad G_c(s) = \frac{\tau_1}{K(\tau_c+\tau_3)}\left[1+\frac{1}{\tau_1 s}\right][\tau_2 s+1]$$

$$= K_c\left[1+\frac{1}{T_i s}\right][T_d s+1]$$

$$\text{Tuning:} \quad K_c = \frac{\tau}{K(\tau_c+\tau_3)} \quad T_i = \tau_1 \quad T_d = \tau_2$$

- Presence of negative lead $\Rightarrow$ smaller gain

- Inverse/overshoot processes:
  less controllable $\Rightarrow$ gain reduction

- **Examples:**
  - Distillation columns
  - Exothermic chemical reactors
  - Result of interaction between control loops

FOPDT Processes

$$\begin{aligned}
\text{Process:} \quad G(s) &= \frac{Ke^{-ds}}{\tau s + 1} \\
\text{Spec:} \quad G_{c\ell}(s) &= \frac{e^{-ds}}{\tau_c s + 1} \\
\text{Controller:} \quad G_c(s) &= \frac{\tau s + 1}{K} \frac{1}{\tau_c s + 1 - e^{-ds}}
\end{aligned}$$

$$\begin{aligned}
\textbf{Case 1:} \quad e^{-ds} &\sim 1 - ds \\
\Rightarrow \quad G_c(s) &= \frac{\tau}{K(\tau_c + d)} \left[1 + \frac{1}{\tau s} \right]
\end{aligned}$$

$$\text{Tuning:} \quad K_c = \frac{\tau}{K(\tau_c + d)} \quad T_i = \tau$$

$$\textbf{Case 2:} \quad e^{-ds} \sim \frac{1-\frac{d}{2}s}{1+\frac{d}{2}s}$$

$$\Rightarrow \quad G_c(s) = \frac{\tau}{K(\tau_c+d)}\left[1+\frac{1}{\tau s}\right]\left[\frac{\left(\frac{d}{2}\right)s+1}{\left(\frac{\tau_c}{\tau_c+d}\right)\left(\frac{d}{2}\right)s+1}\right]$$

$$\text{Tuning:} \quad K_c = \frac{\tau}{K(\tau_c+d)}, \quad T_i = \tau, \quad T_d = \frac{d}{2},$$

$$\alpha = \frac{\tau_c}{\tau_c+d}$$

- ➢ $e^{-ds} \approx 1 - ds \Rightarrow$ degradation of performance
- ➢ Dead time appears in denominator of gain formula
  longer dead time $\Rightarrow$ *smaller gain*
- ➢ A filter formula in derivative unit
- ➢ Filter parameter (α) is *not* adjustable in practice

Selection of τ_c Value

➢ **Min IAE** for disturbance inputs: $\tau_c = 0$

☞ **PI** controller: $0.1 < \frac{d}{\tau} < 0.5$

☞ **PID** controller: $0.1 < \frac{d}{\tau} < 1.5$

➢ **Min IAE** for setpoint inputs: (for $0.1 < \frac{d}{\tau} < 1.5$)

☞ **PI** controller: $\tau_c = \frac{2}{3}d$

☞ **PID** controller: $\tau_c = \frac{1}{5}d$

➢ 5% overshoot on setpoint inputs: $\tau_c = d \Rightarrow K_c = \frac{0.5}{K}\frac{\tau}{d}$

Practical Conclusions

- **Spec:** unity closed-loop gain
  $\Rightarrow$ **I** action as the basic controller mode

- Simplest process $\longrightarrow$ complex $\Longrightarrow$ **I** $\rightarrow$ **PI** $\rightarrow$ **PID**

  Simplest controller $\rightarrow$ complex $\Longrightarrow$ **P** $\rightarrow$ **PI** $\rightarrow$ **PID**

- **I** mode is indicated for very fast processes with
  P mode: compensate for major time constant
  D mode: compensate for 2nd time constant/delay

➢ A simple **tuning procedure**:

- ☞ To set integral time equal to major time constant
- ☞ To set derivative time equal to 2nd time const. or $\dfrac{d}{2}$
- ☞ To adjust controller gain to obtain desired closed-loop response

➢ For *inverse/overshoot* response processes: $K_{c_{max}} = \frac{1}{K}\frac{\tau}{\tau_3}$

➢ For processes with *dead time*: $K_{c_{max}} = \frac{1}{K}\frac{\tau}{d}$

➢ For *integrating* processes:

- ☞ Use **P** mode or **PD** mode ($T_d = \tau$)
- ☞ Offset will result for disturbance inputs

Example: Steam Heater

➢ Temperature control loop of a Heat Exchanger:

$$\text{gain} = 0.8\%/\%; \quad \text{time const.} = 33.8 \text{ sec}; \quad \text{dead time} = 11.2 \text{ sec}$$

➢ Synthesized controller:

$$G_c(s) = \frac{33.8}{0.8(\tau_c + 11.2)} \left(1 + \frac{1}{33.8s}\right) \left\{ \frac{1 + 5.6s}{1 + \frac{5.6\tau_c}{\tau_c + 11.2}} \right\}$$

➢ Integral and derivative times:

$$\begin{aligned} T_i &= \tau = 33.8 \text{ sec} = 0.56 \text{ min} \\ T_d &= \tfrac{d}{2} = \ 5.6 \text{ sec} = 0.093 \text{ min} \end{aligned}$$

➢ **Min IAE** gain for disturbance input:

$$
\begin{aligned}
\tau_c &= 0 \\
\Longrightarrow \qquad K_c &= \frac{33.8}{(0.8)(11.2)} = 3.8\%/\%
\end{aligned}
$$

➢ **Min IAE** gain for setpoint input:

$$
\begin{aligned}
\tau_c &= \frac{d}{5} = 2.24 \text{ sec} \\
\Longrightarrow \qquad K_c &= \frac{33.8}{(0.8)(11.2+2.24)} = 3.1\%/\%
\end{aligned}
$$

➢ 5% overshoot on setpoint input:

$$
K_c = \frac{(0.5)(33.8)}{(0.8)(11.2)} = 1.9\%/\%
$$

➢ **Min IAE** parameters for setpoint inputs:

$$
\begin{aligned}
K_c &= \frac{1.086}{0.8}\left(\frac{11.2}{33.8}\right)^{-0.869} &= 3.5\%/\% \\
T_i &= \frac{33.8}{0.74-0.13(11.2/33.8)} &= 48.5 \text{ sec } (0.81 \text{ min}) \\
T_d &= 0.348(33.8)\left(\frac{11.2}{33.8}\right)^{0.914} &= 4.3 \text{ sec } (0.071 \text{ min})
\end{aligned}
$$

Example: Steam Heater (cont.)

➢ **Process model**: $G(s) = \dfrac{1\ (\%/\%)\ e^{-0.13s}}{0.62s + 1}$ (in min)

➢ **Series PID controllers**:

	$K_c^{'}$ (%/%)	$T_I^{'}$ (min)	$T_D^{'}$ (min)
QDR	5.6	0.27	0.07
Min IAE Disturbance	3.8	0.14	0.08
Min IAE Set point	3.8	0.81	0.06
Synthesis ($\tau_c = 0$)	4.6	0.62	0.07

➢ Response of heater outlet temperature to a 5^oC raise in setpoint

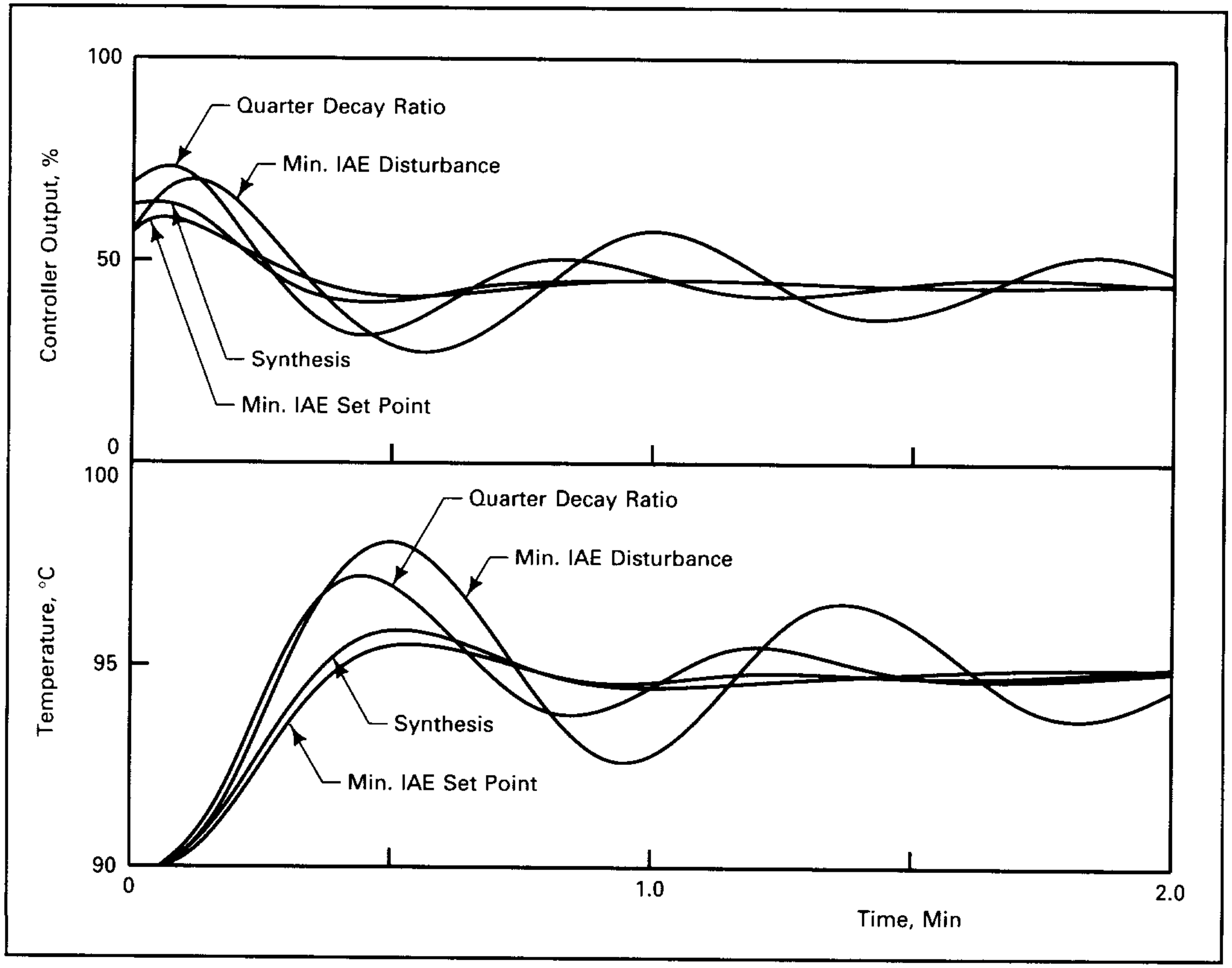


Fig. 5-5. Response of Heater Outlet Temperature to a 5°C Change in Set Point.

➢ The synthesis response and the **min IAE** setpoint response are more conservative than the other two