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The Ferromagnetic Faraday Effect at Microwave Frequencies and its Applications

The Microwave Gyrator

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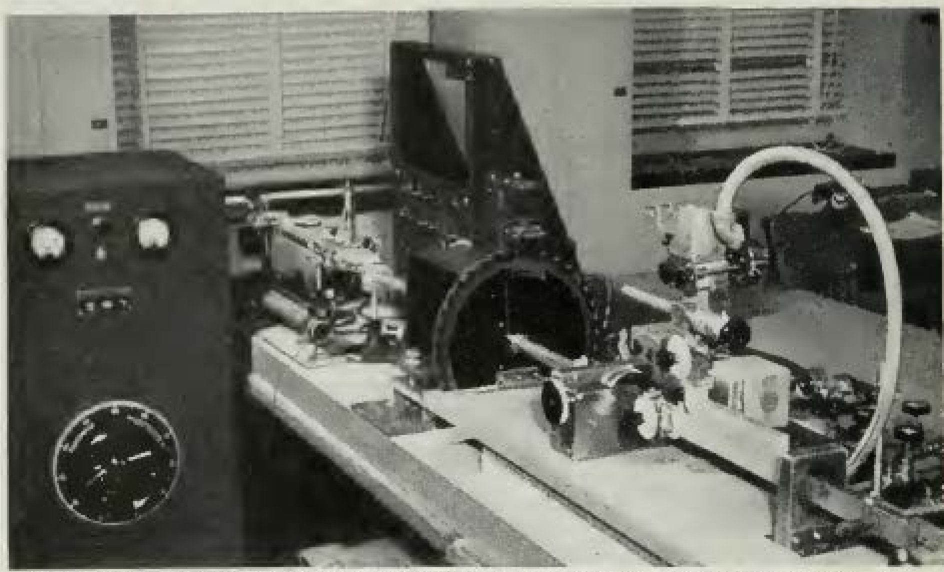
A new microwave circuit element dependent on the Faraday rotation of a polarized wave has been developed. The element violates the reciprocity theorem and, because it shares this property with a gyroscope and because it is dependent on gyromagnetic resonance absorption, it has been termed a microwave gyrator. It is a low-loss broadband device with many applications. Among these are one-way transmission systems, microwave circulators, microwave switches, electrically controlled variable attenuators and modulators.

The microwave gyrator has been realized by making use of the Faraday rotation in pieces of ferrite placed in the waveguide. Polder has previously shown, in his analysis of the gyromagnetic resonance phenomenon, that ferromagnetic substances should show appreciable Faraday rotations at microwave frequencies. In the present study, Polder's analysis has been extended to include a wave being propagated through a ferromagnetic substance with dielectric and magnetic loss, and data are presented which give experimental verification of the theory. In addition an experimental technique is described which may be of some interest in studying the properties of ferrites at microwave frequencies.



Photograph of the experimental setup shown diagrammatically in Fig. 5.

INTRODUCTION

In a recent series of articles, Tellegen[1] has discussed the possible applications of a new circuit element which he calls a gyrator. He defines the ideal gyrator, in principle, as a passive four-pole element which is described by: (see Fig. 1)

$$v_1 = -Si_2 \qquad v_2 = Si_1 \tag{1}$$

Since the coefficients above are of opposite sign, the gyrator violates the theorem of reciprocity. Any network composed of the usual electrical circuit elements—resistors, inductors, capacitors, and transformers—will satisfy the theorem of reciprocity. In simple terms, this theorem states that if one inserts a voltage at one point in the network and measures the current at some other point, their ratio (called the transfer impedance) will be the same if the positions of voltage and current are interchanged. In the gyrator, however, this transfer impedance for one

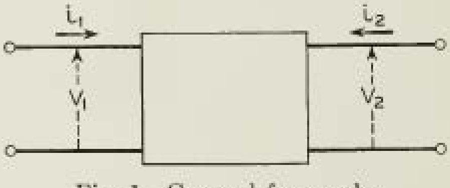


Fig. 1—General four-pole.

direction of propagation is the negative of that for the other direction of propagation. Essentially this means that a 180° phase difference exists between the two directions of propagation. For this reason it has been suggested that the element could be more aptly called a directional inverter.[2]

Network synthesis today is based upon the existence of four basic circuit elements: the capacitor, the resistor, the inductor, and the ideal transformer. It is apparent that the introduction of a fifth circuit element, the gyrator, would lead to considerably improved solutions for many network problems. In fact, Tellegen[1] has shown that the synthesis of resistanceless four-pole networks would be much simplified by its introduction. In addition, McMillan[3] has shown that it would be possible to construct a one-way transmission system if a gyrator were available, and Miles[4] has shown that it would be possible by use of a gyrator to construct a network which is equivalent to a Class A vacuum tube amplifier circuit. While the realizable power gain of these gyrator circuits is necessarily always not greater than unity, many other networks including gyrators are possible which have properties analogous to vacuum tube circuits and some of these may be of practical importance. Since this new element offers such interesting possibilities in network synthesis, a study has recently been made in these Laboratories of possible methods for realizing the gyrator.

A gyrator was employed by Bloch[5] in his measurement of the magnetic moment of the proton. Bloch made use of the phenomenon that if two crossed coils with a mutual core are adjusted so that there is zero mutual inductance between them and if a steady magnetic field is applied perpendicular to the axes of both coils, then an ac voltage applied to one of the coils will induce a voltage in the second due to the gyromagnetic resonance phenomenon. This induction is ordinarily extremely small unless the magnetic field is adjusted so that the exciting frequency coincides with a gyromagnetic resonance frequency of the material which forms the mutual core of the two coils. In Bloch's experiment, the magnetic field was held constant and the exciting frequency was adjusted until it coincided with the gyromagnetic resonance frequency of the proton. If they were wound over a paramagnetic or ferromagnetic material, the two crossed coils would form a gyrator when the magnetic field was adjusted so that the frequency of the exciting field coincided with the gyromagnetic resonance of the unpaired electrons. The fact that this structure constituted a gyrator was first recognized by Tellegen[1] and has been discussed by Beljers and Snoek[6] in a paper which gives a very satisfying physical model with which to interpret gyromagnetic phenomena occurring within ferrites.

Physical analysis indicates that the properties of ferromagnetic materials can be explained by assuming that the electron behaves as if it were a negatively charged sphere which is spinning about its own axis with a fixed angular momentum. This rotation of charge imparts to the electron a magnetic moment which is a function of the electric charge on the electron, the angular velocity of the electron, and its size. Thus the electron behaves as if it were a spinning magnetic top, whose magnetic moment lies along the axis of rotation, and its behavior can be understood by considering a spinning gyroscope suspended in gimbal rings at a point not coinciding with its center of gravity. If a gyroscope, thus supported in a gravitational field, is lifted away from its position of minimum potential energy and then released, it will not return to the position of minimum energy but will precess about the vertical axis. This is illustrated in Fig. 2 where the spinning gyroscope makes an angle θ with the vertical z, axis. Its equilibrium motion, in the absence of damping, is a precessional motion about the vertical axis with a velocity ω_p.

If the gyroscope be regarded as initially hanging vertically downward

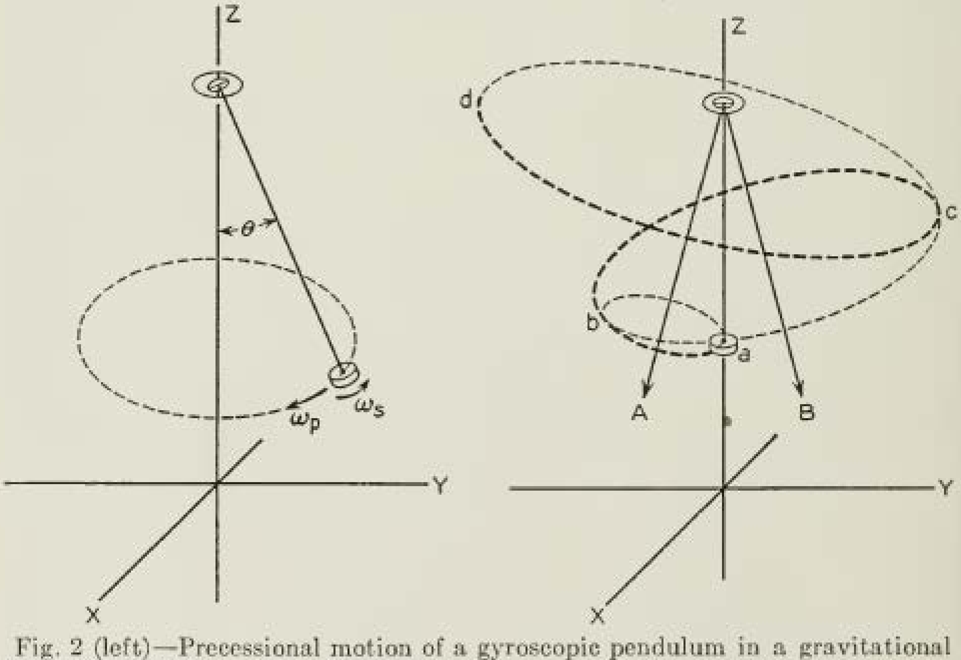


Fig. 2 (left)—Precessional motion of a gyroscopic pendulum in a gravitational field. Fig. 3 (right)—Precessional motion of a gyroscopic pendulum in a gravitational field which oscillates between the directions A and B.

as indicated in Fig. 3 and then a gravitational force is suddenly made to act along the y axis so that the net gravitational force acts along A, it is obvious that the gyroscope will begin to precess about the gravitational field direction as indicated by the small dotted circle. However, if after completing a half cycle, the horizontal component of the gravitational field is reversed so that now the net gravitational field acts along the vector B, the gyroscope will begin to precess about B as indicated by the intermediate size dotted circle. If the horizontal component of the gravitational field is again reversed after the gyroscope completes another half cycle in its precession, the gyroscope will again begin to precess about the direction A and the actual path of the precessional motion will be along the path a-b-c-d. If this process is continued indefinitely, the gyroscope will precess in larger and larger circles around the vertical until the damping becomes large enough to contain the gyroscope in some equilibrium circle (assuming that the damping is large enough to accomplish this).

The above model affords a classical picture which can be used quite readily to describe the motion of the electrons in a ferrite. If the ferrite is initially saturated along the z axis by a steady magnetic field, the electrons will come to rest with their magnetic moments lying along the z axis, as the gyroscope in Fig. 3. If now an alternating magnetic field is applied along the y axis, the electrons will begin to precess in larger and larger circles about the z axis until they finally reach some equilibrium position under the influence of the magnetic fields and the damping. Thus it is apparent in the gyromagnetic resonance experiments described above why an alternating field applied perpendicular to a steady magnetic field in a ferrite will give rise to a varying flux perpendicular to both the steady field and the alternating field. It is also apparent why the alternating flux along the x axis is 90° out of phase with the alternating flux along the y axis. Since precession of the top will always be in the same direction regardless of whether the alternating field is applied along the x or y axes, consideration of Fig. 3 makes it apparent how the two crossed coils with ferrite at their center can constitute a gyrator which violates the reciprocity relation in a manner described by Equations (1). To the present time, however, no practical circuit element making use of this phenomenon has been constructed because the coefficient of coupling between the coils is always small, even in the vicinity of the resonant frequency, and also because the losses in the materials available are so high in the vicinity of resonance that the insertion loss of such a device would be prohibitively large.

McMillan in his original article,[3] showed that a gyrator could be realized by means of mechanically coupled piezo-electric and electromagnetic transducers. Later, McMillan[7] pointed out that a gyrator could be realized by means of the Hall effect in a square plate of bismuth, as was also predicted by Casimir.[7] Another similar possibility would be an electrical-electrical coupling through a gyroscopic link. A gyrator has been built by W. P. Mason of these Laboratories which makes use of the Hall effect in a crystal of germanium.[8] This gyrator showed an insertion loss somewhat higher than the theoretical loss of 12.3 db. R. O. Grisdale of these Laboratories suggested that these losses could be greatly reduced if the same Hall effect principle were applied to a vacuum tube which contained four electrodes which could both emit and collect electrons. This device is no longer passive, but such a structure has been built and showed an insertion loss of about 7 db, only slightly higher than the theoretical loss which would be expected from this geometry.

In view of the substantial losses found to exist in the earlier forms of gyrator discussed above, a study of other "anti-reciprocal" phenomena which might lead to the realization of a relatively low loss gyrator was undertaken.

It has long been known that the Faraday rotation of the plane of polarization in optics is anti-reciprocal. In order to observe the Faraday rotation, polarized electromagnetic waves must be transmitted through a transparent isotropic medium parallel to the direction of the lines of force of a magnetic field. The effect is usually produced by placing the material along the axis of a solenoid. The rotation is "positive" if it is in the direction of the positive electric current which produces the field and "negative" if in the opposite direction. All optically transparent substances show the Faraday rotation.

Its anti-reciprocal property distinguishes the Faraday effect from optical rotations caused by birefringent crystals, or by the Cotton-Mouton effect, which are reciprocal. That is, if a plane polarized lightwave is incident upon a birefringent crystal in such a manner that the plane of polarization is rotated through an angle θ in passing through the crystal, then this rotation will be cancelled if the wave is reflected back through the crystal to its source. In the Faraday rotation, however, the angle of rotation is doubled if the wave is reflected back along its path. Hence, if the length of path through the "active" material is adjusted so as to give a 90° original rotation, the beam on being reflected will have its plane of polarization rotated a total of 180° in passing in both directions through the material. Thus, the Faraday rotation in optics affords an anti-reciprocal relation quite analogous to the anti-reciprocal property of the gyrator postulated by Tellegen.

Lord Rayleigh[9] described a one-way transmission system in optics which makes use of the Faraday rotation. Lord Rayleigh's "one-way" system consisted of two polarizing Nicol prisms (oriented so that their planes of acceptance made an angle of 45° with each other), with the material causing the Faraday rotation placed between them. Thus, light which was passed by the first crystal and whose plane of polarization was rotated 45° would be passed by the second crystal also. But, in the reverse direction, the rotation would be in such a sense that light which was admitted to the system by the second crystal would not be passed by the first.

Although Rayleigh's one-way transmission system can be actually realized, it is experimentally difficult since most substances show extremely small Faraday rotations. In fact, large rotations for transparent substances in the optical region are of the order of one degree per cm path length for an applied magnetic field of 1000 oersteds. To realize a rotation of 45° would require maintaining a field of 1000 oersteds over a distance of approximately one-half meter. The Faraday effect in ferromagnetic substances, however, is unique in that it shows rotations many orders of magnitude greater than the rotations exhibited by any other substances. For instance, König[10] reports rotations of 382,000°/cm by passing light through thin layers of magnetized iron. These data, of necessity, however, were taken on extremely thin sections and the total rotation obtained for any specimen did not exceed 10°. In order to obtain appreciable rotations in a device of practical size, it is necessary to obtain a material which shows a rotation at least intermediate between those reported for iron and other ordinary materials. In addition, in order to make effective use of these rotations, the material must be transparent to the radiation which is being used.

THEORY OF THE FERROMAGNETIC FARADAY EFFECT

Polder[11] has shown in his analysis of the ferromagnetic resonance phenomenon, that a plane electromagnetic wave at microwave frequencies should show appreciable Faraday rotation when propagated through a ferromagnetic material which is magnetized in a direction parallel to the direction of propagation of the wave. Polder has neglected both magnetic and dielectric losses in his analysis and although for the ferrites which are of greatest interest, this approximation is quite valid, nevertheless the more complete theory is developed below. The exact theory of this phenomenon should, of course, be approached through quantum mechanics, but since the classical theory, in this particular case, gives a result as satisfactory as the quantum theory and since it lends itself more aptly to a fundamental physical interpretation of the phenomenon, it is the classical theory which is developed here. Quantum mechanically, Faraday rotations in the optical region are accounted for by the Zeeman splitting of the spectral lines.

The classical model which proves quite adequate for the description of ferromagnetic resonance is that illustrated in Figs. 2 and 3 which regards the electrons of the material which contribute to the magnetism as being spinning magnetic tops. The angular momentum of each electron is:

$$|J| = \tfrac{1}{2}(h/2\pi) \qquad (2)$$

$\vec{J}$ = Angular momentum of electron (gm cm^2/sec)

h = Planck's constant (6.62×10^{-27} erg sec)

The magnetic moment which arises due to this rotation is:

$$|\mu_B| = \frac{eh}{4\pi mc} \qquad (3)$$

where:

$\vec{\mu}_B$ = Magnetic moment of electron (Bohr magneton)

e = Charge on electron (4.80×10^{-10} E.S.U.)

m = Mass of electron (9.10×10^{-28} gm)

c = Velocity of light (3×10^{10} cm/sec)

The so-called gyromagnetic ratio of the electron is the ratio of these quantities and is given by:

$$\gamma = 2\,\frac{e}{2mc} = \frac{|\mu_B|}{|J|} \qquad (4)$$

If a steady magnetic field is applied to the sample such that the electron sees an effective field H, then a torque will be applied to the electron which tries to turn the electron so that its magnetic moment lies along the field direction. However, as indicated in Fig. 2, the electron will precess around the field direction until damping forces dissipate the energy of precession. The equation of motion of the electron is:

$$\vec{\mu}_B \times \vec{H} = \frac{d\vec{J}}{dt} = \gamma^{-1}\frac{d\vec{\mu}_B}{dt} \qquad (5)$$

The equation of motion of the magnetization per unit volume can thus be written:

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \vec{H} \tag{6}$$

where:

$\vec{M}$ = Magnetization of medium

$\vec{H}$ = Macroscopic internal magnetic field

The above equation, however, does not include damping. The damping force, regardless of its origin, must be so introduced into the above equation that it tends to cause the electron's axis of rotation to line up with the field direction. It has been shown by Yager, Galt, Merritt and Wood[12] that the shape of the resonance absorption line can be accounted for if the damping term is introduced in the following way:

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \vec{H} - \frac{\gamma\alpha}{|M|}[\vec{M} \times (\vec{M} \times \vec{H})] \tag{7}$$

The vector $\vec{M} \times (\vec{M} \times \vec{H})$ is simply a vector which is in the proper direction to act as a damping force (torque) and the coefficient is chosen so as to give the correct units along with the parameter, α, which must be determined experimentally and which gives the magnitude of the damping torque.

Equation 7 then is the equation of motion of the magnetization of an arbitrarily shaped body under the action of an arbitrary internal field, H. In the appendix, it is shown that if a steady magnetic field, H_a, is applied along the z axis and then a small alternating field is applied in an arbitrary direction to a sample which is infinite in size, the equation relating the resulting alternating flux density, b, and the applied alternating field, h is:

$$\begin{aligned} b_x &= \mu h_x - jKh_y \\ b_y &= jKh_x + \mu h_y \\ b_z &= h_z \end{aligned} \tag{8}$$

where

$$\mu = \mu' - j\mu'' \tag{9}$$

$$K = K' - jK'' \tag{10}$$

Equations which give μ and K in terms of the applied magnetic field and fundamental atomic constants are given in the appendix.

Equations (8) are easily interpreted in terms of the spinning gyroscope model of Fig. 3. If magnetic losses had been ignored (i.e. $\alpha = 0$) then both μ and K would have been real. Under this condition, it is seen that if an alternating field, h_y, is applied along the y axis, then an alternating flux, b_y, is created along the y axis which is in phase with h_y, and an alternating flux, b_x, is created which is 90° out of phase with h_y. Reciprocity between the x and y directions would demand that both terms containing jK should have the same sign. Thus, Equations (8) give a quantitative expression for the results which were previously qualitatively deduced by means of the electronic model illustrated in Figs. 2 and 3.

If a waveguide is filled with a ferromagnetic material such as a ferrite and if then a steady magnetic field is applied along the axis of the waveguide, it is necessary in order to describe this wave to find a solution to Maxwell's equations which is consistent with Equations (8) and in which b, h, E and D are all proportional to exp $[j\omega t - \Gamma z]$. This problem is not solved exactly. However, in the appendix a solution is obtained for an infinite plane wave. It is found that the ferromagnetic medium can support only a positive or a negative* circularly polarized wave or a combination of both. It is also shown in the appendix that the propagation constants for these two circularly polarized waves are different and are given by the following expressions:

$$\Gamma_+ = \frac{j\omega}{c} \sqrt{(\mu + K)[\varepsilon]} \tag{11}$$

and

$$\Gamma_- = \frac{j\omega}{c} \sqrt{(\mu - K)[\varepsilon]} \tag{12}$$

where

$\Gamma_\pm$ = Propagation constant

ω = Angular frequency of wave

c = Velocity of light in unbounded space (3×10^{10} cm/sec)

ε = Complex dielectric constant of medium

In Equations (11) and (12) it is apparent that the effective permeability of the medium to a positive circularly polarized wave, for in-

* The usual notation is used here, where the positive component is the component which is rotating in the direction of the positive electric current which creates the steady longitudinal field.

stance, is given by the expression $(\mu + K)$, and not by the usual permeability, $b_z/h_z = \mu$. It is also apparent that the quantity $\mu + K$ can vary over wide limits in the vicinity of the ferromagnetic resonance. For this reason, care must be taken in interpreting permeability data for ferromagnetic materials which now occur in the literature and which were obtained by means of impedance measurements at microwave frequencies, since the above equations indicate that this method does not measure the same quantity that is measured at low frequencies by means of a toroidal sample overwound with two coils. The low frequency measurement of permeability obviously measures the quantity which is designated as μ in Equation (8).

If Equations (11) and (12) are solved for the attenuation constants, $\alpha_\pm$, and the phase constants, $\beta_\pm$, the following results are obtained:

$$\alpha_\pm = \frac{\omega}{c}\sqrt{\frac{(\mu' \pm K')\varepsilon'}{2}} \cdot \left[\left\{\sqrt{(1 + \tan\delta_m[4\tan\delta_d + \tan\delta_m(1 + \tan^2\delta_d)] + \tan^2\delta_d}\right\} \cdot - 1 - \tan\delta_m \tan\delta_d\right]^{\frac{1}{2}} \tag{13}$$

and

$$\beta_\pm = \frac{\omega}{c}\sqrt{\frac{(\mu' \pm K')\varepsilon'}{2}} \cdot \left[\left\{\sqrt{(1 + \tan\delta_m[4\tan\delta_d + \tan\delta_m(1 + \tan^2\delta_d)[+ \tan^2\delta_d}\right\} \cdot + 1 + \tan\delta_m \tan\delta_d\right]^{\frac{1}{2}} \tag{14}$$

where:

$$\tan\delta_m = \frac{\mu'' \pm K''}{\mu' \pm K'}$$

(The + sign must be used for a positive circularly polarized wave; the negative sign for the negative circularly polarized wave.)

$$\tan\delta_d = \frac{\varepsilon''}{\varepsilon'} = \text{dielectric loss tangent}$$

$$\varepsilon = \varepsilon' - j\varepsilon'' = \text{complex dielectric constant}$$

It is almost impossible to get a feeling for what these equations mean with respect to a wave travelling through the medium, especially since μ and K are given by equations which are almost as difficult to perceive. An appreciation of these equations can be obtained however, by reference to Fig. 4 which gives qualitatively the behavior predicted by these expressions. Essentially, α and β are functions of two variables. These are ω, the frequency of the wave, and H_a, the applied magnetic field. In Fig. 4, the index of refraction and attenuation of the positive circularly polarized component are given relative to these values for

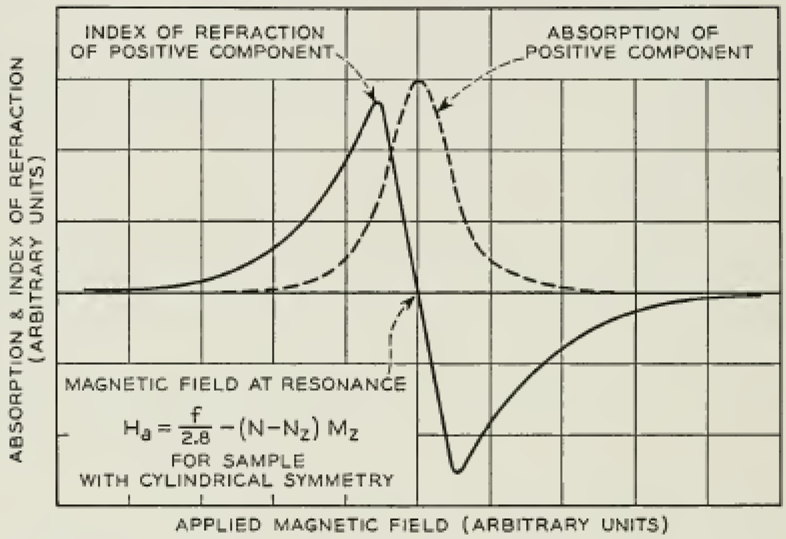


Fig. 4—Index of refraction and absorption of a positive circularly polarized wave relative to the same quantities for a negative circularly polarized wave being propagated through a magnetized medium.

the negative component. Hence, both the index of refraction and attenuation of the negative component are represented by the abscissa of the graph. In Fig. 4 these quantities are plotted as a function of the applied magnetic field for a wave of a fixed frequency. Many of the properties of the medium are clearly displayed in this graph. In particular, as the field necessary for ferromagnetic resonance is approached, the attenuation of the positive component becomes larger and larger. Eventually this component will be substantially completely absorbed and only the negative circularly polarized component will be propagated. Hence it should be possible to establish a circularly polarized wave in a waveguide simply by passing the dominant mode through a ferromagnetic material which is subjected to a longitudinal magnetic field of the proper amplitude. However, there will be an absorption of one-half of the power being propagated. If Fig. 4 had been plotted as a function

of the frequency of the wave for a fixed magnetic field, a similar set of curves would have resulted. This set would indicate the frequency dependence of the Faraday rotation. If the frequency of the wave is far removed from the resonance frequency, the difference between the indices of refraction of the positive and negative component is not frequency dependent. However, near resonance, this difference is a very rapidly varying function of the frequency. It is to be remembered that these equations were derived for an infinite plane wave. However, it would be expected that these equations would describe quite accurately the propagation of the dominant mode in a waveguide. The approximation would, of course, be better when the cut-off wavelength was much greater than the unbounded wavelength. This condition is met when the waveguide is filled with ferrite and for these cases quantitative agreement is obtained.

The above analysis shows that if a dominant mode wave (plane polarized) is incident upon a ferromagnetic material which is magnetized along the length of the waveguide, the wave will split into positive and negative circularly polarized waves whose phase constants are given by Equation (14).) Since the two circular components travel with different velocities in the medium, they will upon emerging from it unite to form a plane polarized wave whose plane of polarization has been rotated with respect to the incident polarization. The angle of rotation of the polarization is given by:

$$\theta = \frac{\ell}{2}\left[\beta_{-} - \beta_{+}\right] \tag{15}$$

where:

ℓ = path length through ferromagnetic material (cm)

In order to evaluate Equation (15), it must be combined with Equation (14). However, a few approximations are valid in Equation (14) which make it much simpler. In particular many ferrites exist for which the magnetic losses are extremely small as long as the internal field within the body is kept small so that the frequency of the wave does not approach the ferromagnetic resonance frequency. This field can be kept small if the magnetic field is not raised above the point necessary to saturate the ferrite. Kittel has shown that for a finite body the effective internal magnetic field that determines the resonant frequency is given by:

$$H_e^2 = [H_a + (N_x - N_z)M_z][H_a + (N_y - N_z)M_z]$$

where H_e is in oersteds. The ferromagnetic resonance frequency is given by:

$$f_{res} = 2.8H_e \sqrt{1 + \alpha^2} \text{ megacycles} \tag{16}$$

It is easily shown that the following formula is approximately valid for a ferromagnetic body with a circular or square cross-section

$$H_e = \frac{4\pi + N(\mu - 1)}{4\pi + N_z(\mu - 1)} H_a \tag{17}$$

at saturation. Where:

μ = true dc permeability at saturation

N = demagnetizing factor in x and y directions.

If an average value of 1000 is assumed for the dc permeability, then H_e can be readily computed for various shapes.

For a thin disc:

$$N = 0 \qquad N_z = 4\pi$$

and

$$H_e = \frac{H_a}{1000}$$

If a thin disc saturates at 1500 gauss, then:

$$H_e = 1.5 \text{ oersteds}$$

$$f_{res} \approx 4.2 \text{ megacycles}$$

For a long thin pencil:

$$N_z = 0 \qquad N = 4\pi$$

and

$$H_e = 1000H_a$$

For this case the body could be saturated with a field of about 1.5 oersteds, so:

$$H_e = 1500 \text{ oersteds}$$

and

$$f_{res} = 4200 \text{ megacycles}$$

If, for this case, the resonance frequency is so close to the operating frequency that losses due to ferromagnetic resonance become prohibitive, it is wise to then raise the applied field to some high value, so that the resonance frequency will fall well above the operating frequency. Thus, for many cases of interest it is possible by various means to place the ferromagnetic resonance absorption frequency sufficiently far from the operating frequency so that magnetic losses due to this phenomenon are negligible.* The data accumulated to date indicate that the major component of the magnetic losses at microwave frequencies is due to this phenomenon. Only in a few cases have data been taken which have indicated that other factors, such as domain wall relaxation, contribute to the magnetic loss at microwave frequencies.

If then, the magnetic field is controlled so that the ferromagnetic resonance absorption is negligible, Equations (13) and (14) can be simplified to:

$$\alpha_{\pm} = \frac{\omega}{c}\sqrt{\frac{(\mu' \pm K')\varepsilon'}{2}} \quad \sqrt{\sqrt{1 + \tan^2\delta_d} - 1} \tag{18}$$

and:

$$\beta_{\pm} = \frac{\omega}{c}\sqrt{\frac{(\mu' \pm K')\varepsilon'}{2}} \quad \sqrt{\sqrt{1 + \tan^2\delta_d} + 1} \tag{19}$$

which can be written as:

$$\alpha_{\pm} = \frac{\omega}{c}\sqrt{\frac{|\varepsilon| - \varepsilon'}{2}} \quad \sqrt{\mu' \pm K'} \tag{20}$$

and

$$\beta_{\pm} = \frac{\omega}{c}\sqrt{\frac{|\varepsilon| + \varepsilon'}{2}} \quad \sqrt{\mu' \pm K'} \tag{21}$$

where μ' and K' are given in the appendix.

If Equation (21) is now inserted into Equation (15) a formula for rotation is obtained which is valid within the limits of the above approximations. If in addition, the frequency of the wave is sufficiently greater than the resonance frequency, so that:

$$\omega_{res} \ll \omega \tag{22}$$

then Equation (15) takes the particularly simple form:

$$\frac{\theta}{l} = \frac{\omega}{2c}\sqrt{\frac{|\varepsilon| + \varepsilon'}{2}}\left[\sqrt{1 + \frac{4\pi M_z\gamma}{\omega}} - \sqrt{1 - \frac{4\pi M_z\gamma}{\omega}}\right] \tag{23}$$

Most ferrites saturate at 2,000 gauss or less. Hence, for a frequency of

* This is not always possible, for some ferrites, in the polycrystalline state, exhibit extremely broad ferromagnetic resonance absorption lines and it is difficult to operate at any frequency without appreciable absorption.

9,000 megacycles,

$$\frac{4\pi M_z\gamma}{\omega} \leqq \frac{2000 \times 17.6 \times 10^6}{9000 \times 2\pi \times 10^6} = 0.622 \qquad (24)$$

Hence, the following approximation will be valid to within 5 per cent.

$$\sqrt{1 \pm \frac{4\pi M_z\gamma}{\omega}} \approx 1 \pm \frac{1}{2}\left(\frac{4\pi M_z\gamma}{\omega}\right)$$

With this approximation, Equation (23) reduces to:

$$\frac{\theta}{\ell} = \frac{1}{2c}\sqrt{\frac{|\varepsilon| + \varepsilon'}{2}}\,[4\pi M_z\gamma] \qquad (25)$$

Equation (25) is quite remarkable. Not only does it predict large rotations, but it also predicts that, within the above approximations the rotation will not depend upon the frequency of the incident radiation. For the assumed values,

$$\varepsilon' = 15$$

$$\varepsilon'' = 0$$

$$4\pi M_z = 1000,$$

Equation (25) predicts rotations of,

$$\frac{\theta}{\ell} = 65°/\text{cm}.$$

DESCRIPTION OF EQUIPMENT AND MEASURING TECHNIQUES

The Faraday rotation has been measured in a large number of ferrites in order to verify the above theory and in an effort to improve the characteristics of the microwave gyrator. A diagram of the experimental equipment is given in Fig. 5, and a diagram of the test chamber in which the rotations were measured is given in Fig. 6. In the test chamber, two rectangular waveguides are separated by a circular waveguide, the proper nonreflective transitions being made at each end of the circular section, which is about twelve inches long. One rectangular guide is supported so that it can be rotated about the longitudinal axis of the system. The dominant TE_{10} mode is excited in one rectangular guide, and by means of the smooth transition this goes over into the dominant TE_{11} mode in the circular guide. The rectangular guide on the opposite end will accept only that component of the polarization which coincides with the TE_{10} mode in that guide, the other component being reflected at the transition. Absorbing vanes, inserted in the circular section, absorb this reflected component. The circular guide is placed in a solenoid to establish an axial magnetic field along its length.

The ferrite cylinders to be measured were placed at the mid-section of the circular guide. When a cylinder was used which did not fill the cross-section of the guide, it was supported along the axis of the guide by means of a hollow polystyrene cylinder which did fill the guide.

In addition to measuring the Faraday rotation, measurements of insertion loss were made by determining the power transmitted under identical conditions with the ferrite cylinder removed, and the ellipticity of the transmitted wave was determined by measuring the power transmitted when the rectangular guide on the detector side was rotated to both positions of maximum and minimum transmission. Power transmission measurements could be repeated within 0.2 db. Measurements of the angle of rotation of the plane of polarization could be repeated within $\frac{1}{2}°$ except in the region close to the gyromagnetic resonance where rotations were large and ellipticity so great that it was difficult to decide the positions of maximum and minimum transmission. These errors increased up to the point where the transmitted wave was circularly polarized where it was impossible to measure the angle of rotation.

EXPERIMENTAL RESULTS

Equation (25) indicates that the rotation per unit path length through the ferromagnetic material is proportional to the magnetization of the

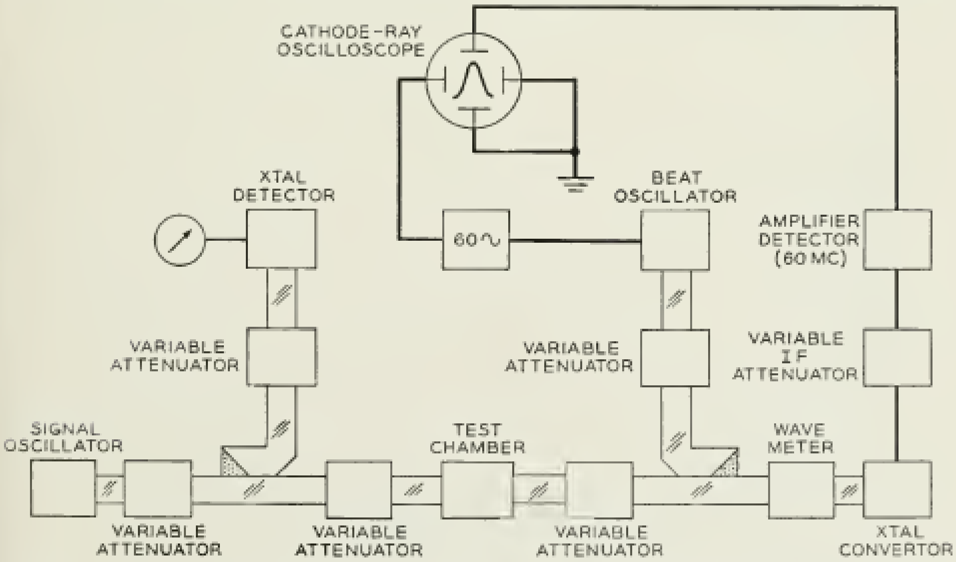


Fig. 5—Experimental equipment set-up used to measure Faraday rotations.

at the transition. Absorbing vanes, inserted in the circular section, absorb this reflected component. The circular guide is placed in a solenoid to establish an axial magnetic field along its length.

The ferrite cylinders to be measured were placed at the mid-section of the circular guide. When a cylinder was used which did not fill the cross-section of the guide, it was supported along the axis of the guide by means of a hollow polystyrene cylinder which did fill the guide.

In addition to measuring the Faraday rotation, measurements of insertion loss were made by determining the power transmitted under identical conditions with the ferrite cylinder removed, and the ellipticity of the transmitted wave was determined by measuring the power transmitted when the rectangular guide on the detector side was rotated to both positions of maximum and minimum transmission. Power transmission measurements could be repeated within 0.2 db. Measurements of the angle of rotation of the plane of polarization could be repeated within $\frac{1}{2}°$ except in the region close to the gyromagnetic resonance where rotations were large and ellipticity so great that it was difficult to decide the positions of maximum and minimum transmission. These errors increased up to the point where the transmitted wave was circularly polarized where it was impossible to measure the angle of rotation.

EXPERIMENTAL RESULTS

Equation (25) indicates that the rotation per unit path length through the ferromagnetic material is proportional to the magnetization of the

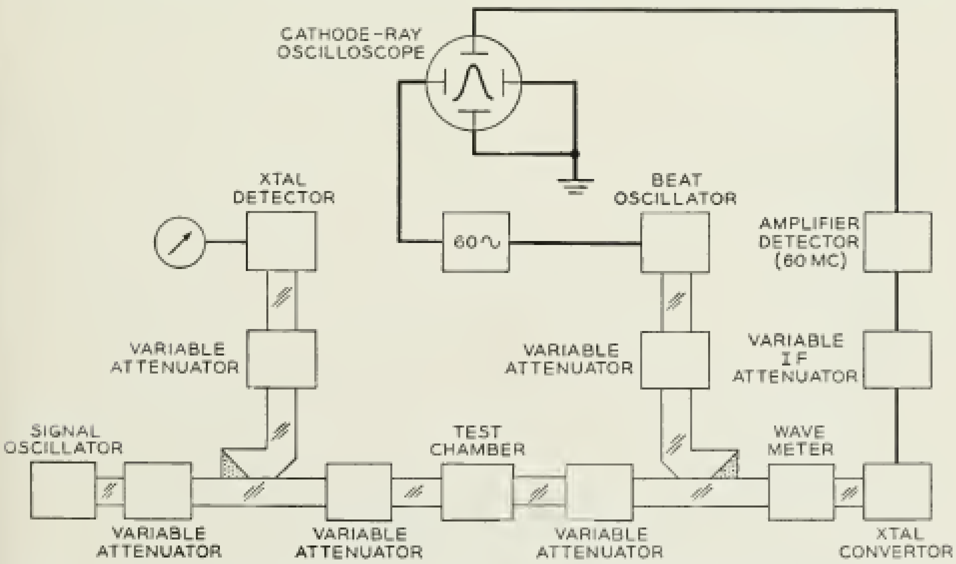


Fig. 5—Experimental equipment set-up used to measure Faraday rotations.

sample and is not dependent directly on the applied magnetic field. Fig. 7 shows the dependence of rotation upon magnetization for a sample of manganese zinc ferrite, and indicates that after the sample is saturated, the rotation is sensibly independent of the applied magnetic field. In addition, the complex dielectric constant and the saturation magnetization of this sample were measured. From these the rotation per centimeter path can be computed from the above theory using

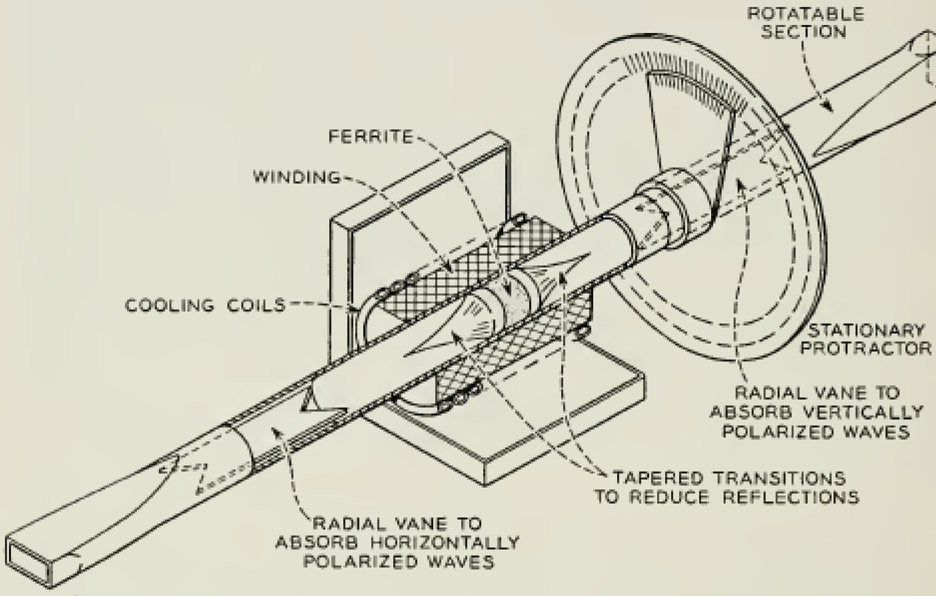


Fig. 6—Detail of test chamber in which rotations were measured.

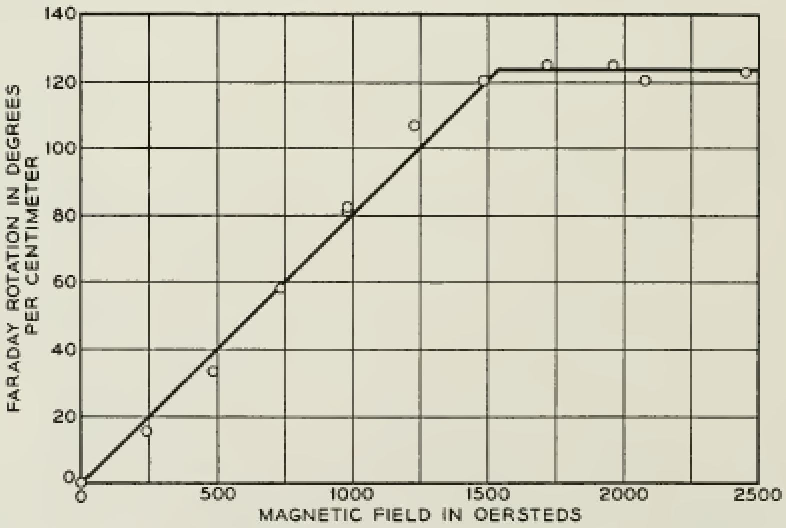


Fig. 7—Angle of rotation versus applied magnetic field for a thin disc of manganese zinc ferrite.

Equation (25). For a particular sample of manganese zinc ferrite, the following measurements were made:

(Sample No. 1)

$$\varepsilon' = 17$$

$$\varepsilon'' = 24$$

$$4\pi M_{sat} = 1500 \text{ gauss}$$

Using this data, equation (25) predicts:

$$\frac{\theta}{\ell} = 121.2°/\text{cm}.$$

It is seen in Fig. 6, that the actual measured rotation at saturation is approximately 123°/cm. Hence an extremely good agreement with theory has been obtained for this particular sample.

Equation (25) also indicates that the rotation per unit path length should be sensibly independent of frequency within the above approximations. The data are shown in Fig. 8. However, it will be noticed that the frequency difference between these two sets of data is relatively small (3 per cent), and the cumulative experimental error in measuring angles is such that it is difficult to state that the rotation is closer than 1° between the two sets of data. This represents a possible difference of 5 per cent in the rotation for a change of 3 per cent in the frequency. Thus, even though these preliminary data support Equation (25), it cannot be accepted as conclusive evidence until more measurements can be made over a wider band width.

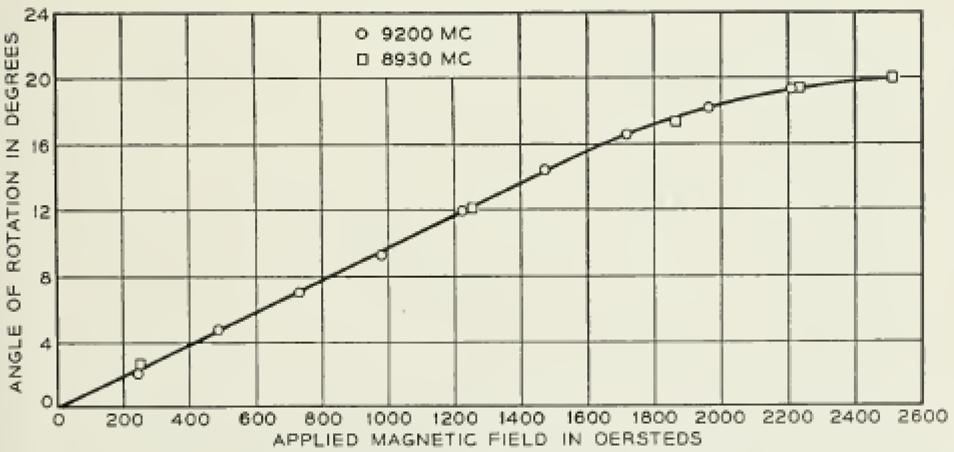


Fig. 8—Dependence of Faraday rotation upon frequency.

The loss characteristics of different ferrites as a function of the applied magnetic field differed distinctly from each other. Some ferrites, such as manganese zinc ferrite showed extremely high loss which was associated with the imaginary part of the dielectric constant. This loss was not affected by the application of a magnetic field but remained substantially constant as the field was applied. However, as the field approached that necessary for ferromagnetic resonance, the total power absorbed by the ferrite increased, since the positive circularly polarized component was almost completely absorbed by the sample. In fact by measuring the ellipticity of the transmitted wave, it is possible to compute the difference between the absorption of the positive and negative circularly polarized components. This has been done for Sample No. 1 and the result is indicated in Fig. 9. If the curve were continued to higher fields, it would represent the shape of the ferromagnetic resonance absorption line.

Some ferrites, such as Ferramic G, showed an almost zero dielectric loss but on the other hand caused an extremely large absorption at 9000 megacycles due to magnetic losses. The major contributions to magnetic loss at this frequency should be either losses associated with a domain wall relaxation or ferromagnetic resonance absorption due to anisotropy fields. Unequivocal data can be obtained by the above techniques to identify which loss is predominant. If the loss were due to domain wall relaxation (or resonance) it would absorb both the negative and positive circularly polarized components equally. Thus as the magnetic field was applied and as the ferrite became saturated, the losses in both components should decrease as the domain walls disappeared. However,

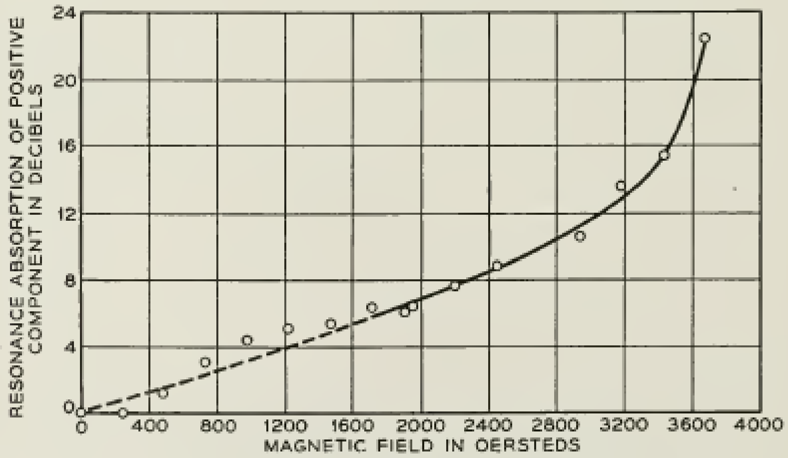


Fig. 9—Ferromagnetic resonance absorption curve determined by measuring the ellipticity of a wave transmitted through a cylinder of ferrite in a waveguide.

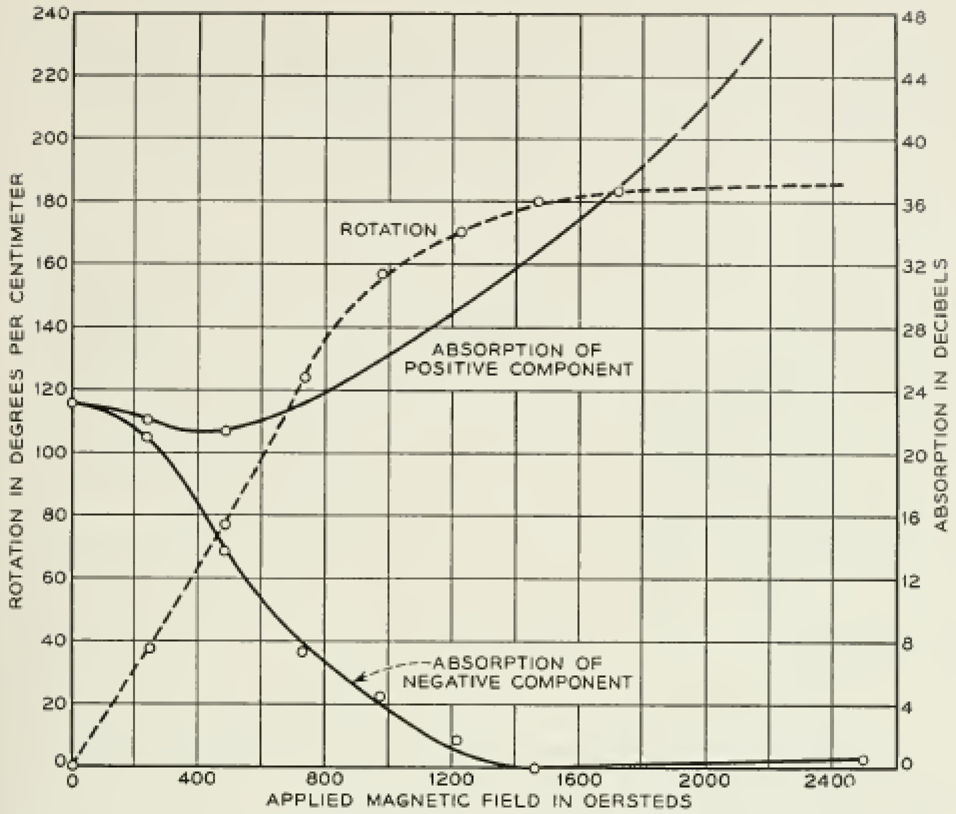


Fig. 10—Rotation of the plane of polarization and absorption of the positive and negative circularly polarized components when an electromagnetic wave (X-band) is propagated through a cylinder of Ferramic G.

if instead the loss is associated with ferromagnetic resonance absorption, the absorption of the positive component should begin to increase as soon as more domains are lined up in a direction where they can absorb the positive component. Thus even before the sample is saturated the absorption of the positive component should be much larger than the absorption of the negative component. Of course, in a polycrystalline sample with a large anisotropy both components can be absorbed by ferromagnetic resonance absorption when the sample is not completely saturated, since the random orientation of the domains which occurred in zero field has not been completely eliminated until complete saturation occurs. Fig. 10 illustrates the rotation per cm path length versus applied magnetic field for a sample of Ferramic G. Superimposed on the same figure are curves showing the absorption of the negative and positive circularly polarized components. It will be noticed that as soon as the sample is saturated, the sample becomes transparent to the negative component but almost completely absorbs the positive component.

Hence the transmitted wave at this point is almost completely circularly polarized, even though the applied magnetic field would indicate that the resonance absorption frequency was far removed from 9000 mc.

Table I gives the data taken on several ferrites at 9000 mc.

APPLICATIONS OF THE FERROMAGNETIC FARADAY EFFECT—THE MICROWAVE GYRATOR

As pointed out in the introduction, the Faraday rotation affords an anti-reciprocal phenomenon from which a microwave gyrator can be constructed. Such a gyrator is illustrated in Fig. 11 along with diagrams which help explain its action. Beneath the gyrator are construction lines which indicate the plane of polarization of a wave as it travels through the gyrator in either direction. On each diagram is a dotted sine wave which is for reference purpose only and indicates the constant plane of polarization of an unrotated wave. It is noticed that for propagation from left to right in Fig. 11, the screw rotation introduced by the twisted rectangular guide adds to the 90° rotation given to the wave by the ferrite element making a total rotation of 180°. For a wave travelling in the reverse direction, these two rotations cancel each other, producing a net zero rotation through the complete element. The unique property of the Faraday rotation becomes immediately apparent from this diagram. In the case of the rotation induced by the twisted rectangular guide, the wave rotates in one direction in going from left to right through the twisted section, and rotates in the opposite direction when it transverses the section from right to left. For the case of the rotation induced by the ferrite element, the direction of rotation is indicated by the arrow in the upper figure for either direction of propagation. The important characteristic of the element is the time phase relation between two points such as A and B in the upper diagram. It is seen with the help of the diagrams illustrating the rotating waves that the field variations are in phase at points A and B for propagation from left to right, and they are 180° out of phase for propagation from right to left. In other words the transmission line is an integral number of wavelengths long between A and B for propagation from left to right and is an odd integral number of half wavelengths long for propagation from right to left.

From the above description of the properties of the gyrator, many of its applications in microwave technology become immediately apparent. Before discussing these applications in more detail, however, it is advantageous to introduce standardized terminology and circuit symbols which apply to the gyrator and to other circuit elements derivable from it.

TABLE I

Sample Number	Material	Dimension (cm)	Applied Magnetic Field (oersteds)	Rotation/cm path	Insertion Loss (db)	Ellipticity* (db)	SWR on Input Line (db)
1	BTL	0.447 × 2.28	0	0	10.0	>50	
	$Mn_\delta Zn_{1-\delta}Fe_2O_4$	(length × dia.)	245	15.6	10.3	>50	
			490	33.5	10.0	23.2	
			735	58.2	9.2	15.0	
			980	81.6	9.1	12.1	
			1225	107	9.2	10.9	
			1470	120	10	10.4	
			1715	125	11	9.3	
			1960	123	11.2	9.0	
			2206	121	11.3	7.7	
			2450	123	11.4	6.6	
			2695	—	12.4	5.0	
			2940	—	13.0	3.7	
			3185	—		3.0	
			3675	—		1.4	
2	BTL		0	0	0.8	>40	
	$Ni_\delta Zn_{1-\delta}Fe_2O_4$	1.36 × 2.28	245	25	1.9	≈40	
			490	44	2.7	≈40	
			735	56	2.9	≈40	
			980	61	2.7	40	
			1225	68	2.8		
			1715	82	3.3		
			1960	85	4.9		
			2450	118	7.3	0.8	
3	Ferramic A	2.54 × 0.635	0	0	1.1	>50	0.7
			245	34.9	0.8	"	0.3
			490	43.7	0.8	"	0.3
			735	48.3	0.8	"	0.3
			980	51.1	1.0	"	0.4
			1225	54.0	1.1	"	—
			1715	57.0	1.1	"	—
			1960	60.0	1.9	"	—
			2450	63.0	3.0	35	—
			2695	64.2	3.7	—	—
4	Ferramic G	1.77 × 2.28	0	0	23.2	≫30	
			245	38	21.4	23.0	
			490	77	16.7	7.6	
			735	124	12.4	2.1	
			980	157	9.9	1.4	
			1225	170	7.7	0.7	
			1470	180	6.0	0.7	
			3430	c.p.	7.1	0.0	

* Data given is the difference in db between the major and minor components of the elliptically polarized transmitted wave.

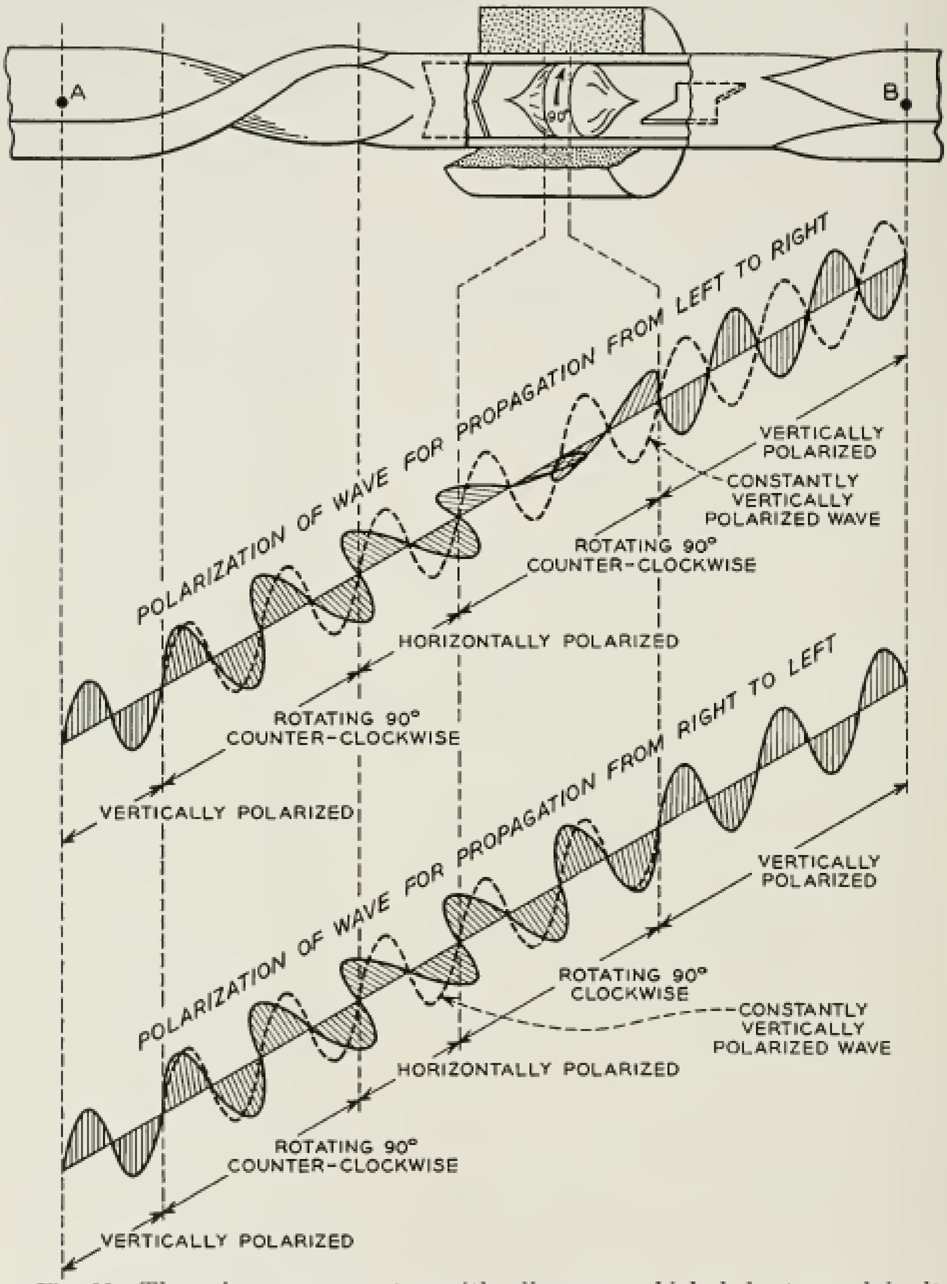


Fig. 11—The microwave gyrator with diagrams which help to explain its operation.

The "active" element of the device, the ferrite cylinder, has been termed a "Faraday Plate."

As was pointed out earlier, the fundamental property of the gyrator is the 180° phase difference introduced between the two directions of propagation through it. Thus the gyrator may be thought of as a four terminal circuit element having no phase shift for one direction of transmission, and having a 180° phase shift for the opposite direction of transmission. A convenient circuit symbol for the gyrator, which indicates this property, is shown in Fig. 12.

If the rectangular waveguides on each side of the Faraday Plate are rotated about their common axis so as to make an angle of 45° with

Fig. 12—Circuit symbol for gyrator.

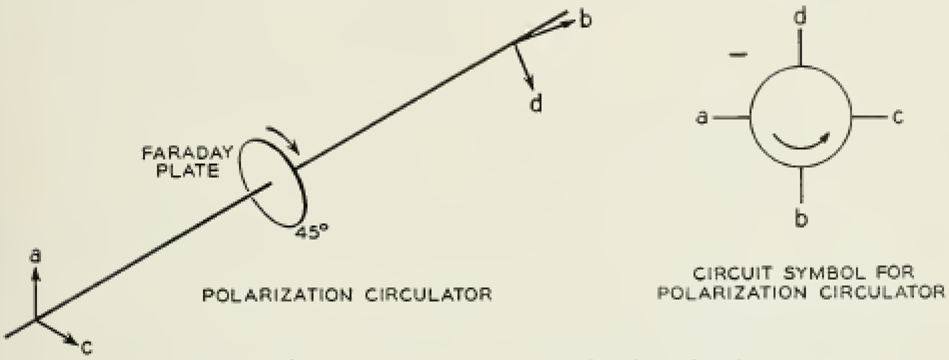


Fig. 13—Schematic diagram of polarization circulator.

each other, then a one-way transmission system can be created which is similar to Lord Rayleigh's one-way transmission system of optics, but with the important difference that this one-way transmission system does not depend upon frequency but is broad band. This one-way transmission system can be used, for example, to isolate the generator or detector from the waveguide in microwave systems. In this application it has the great advantage over the attenuators which are presently used for this purpose in that it can be made practically lossless for the direction of propagation which is desired but the reflected wave will be completely absorbed and hence more complete isolation can be effected.

A more complex and more useful circuit element, than this simple one-way transmission property would at first indicate, is obtained by adding a second connection on each side of the 45° Faraday Plate. It is suggested that this device be called a *polarization circulator*. Thus, the polarization circulator actually has four output branches corresponding to the two different polarizations at each end. The polarizations of the four output branches are indicated in Fig. 13. It is noticed that power sent into the polarization circulator with polarization *a* is turned into polarization *b*, also *b* is turned into *c*, *c* is turned into *d*, and *d* is turned into *minus a*. This property is indicated very clearly by the circuit symbol suggested in Fig. 13, the phase inversion between arms *d* and *a* being indicated by the minus sign between the *d* and *a* arms.

Another one-way transmission system can be created by combining the gyrator with two-normal hybrids. This combination is indicated in Fig. 14. Since this device has all of the fundamental properties of the

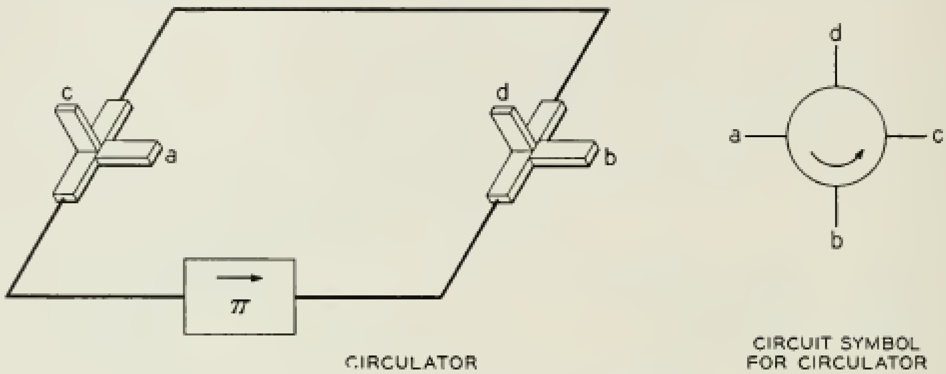


Fig. 14—Schematic diagram of circulator.

polarization circulator with the exception of the phase inversion between arms *d* and *a* it is suggested that it be called a "circulator" and the circuit symbol suggested which indicates its properties is also given in Fig. 14.

This list of applications is obviously not complete since it includes only the fundamental elements from which innumerable specific applications can be made.

In addition to the applications discussed above, which depend upon the anti-reciprocal property of the element for their operation there are several simple applications which are based only upon the fact that the amount of rotation can be controlled externally by adjusting the magnetic field. Among these uses are electrically controlled attenuators, modulators, and microwave switches.

ACKNOWLEDGMENTS

The author is indebted to a number of persons for aid in developing this circuit element. In particular, he wishes to thank A. G. Fox for permission to use his terminology and circuit symbols, and also for the many discussions concerning the properties and uses of these microwave circuit elements. The author is also indebted to S. E. Miller for help in designing the microwave elements and to J. K. Galt for many discussions concerning the theoretical aspects of this paper. The author also wishes to extend his appreciation to J. L. Davis whose able technical assistance made possible the accumulation of much of the data presented in this paper.

APPENDIX

The equation of motion of the magnetization of a ferromagnetic material is:

$$\frac{\partial \vec{M}}{\partial t} = \gamma(\vec{M} \times \vec{H}) - \frac{\gamma\alpha}{|M|}[\vec{M} \times (\vec{M} \times \vec{H})] \qquad (1)$$

where

$\vec{H}$ = internal magnetic field (oersteds)

$4\pi\vec{M}$ = magnetization of medium (gauss)

α = parameter which measures the magnitude of the damping force on the precessing dipole moment of the sample

γ = gyromagnetic ratio of the electron ($\gamma = ge/2mc$ where g is the Landé g factor for the electron).

If a ferromagnetic material is subjected to a steady magnetic field, H_a, along the z axis and if then an alternating field is applied in an arbitrary direction, Equation (1) must be solved in order to find the behavior of the magnetization of the material. To solve this problem, the following notation is introduced:

$4\pi M_z$ = magnetization of medium in absence of alternating field

H_a = externally applied steady magnetic field (oersteds)

h_x, h_y, h_z = components of applied alternating magnetic field

m_x, m_y, m_z = alternating components of magnetization

h_x^i, h_y^i, H_z^i = components of internal magnetic field

$h_x^i = h_x - N_x m_x$

$h_y^i = h_y - N_y m_y$

$H_z^i = H_a + h_z - N_z(M_z + m_z)$

N_x, N_y, N_z = demagnetizing factors of body.

Hence the magnetic field, H, occurring in Equation (1) is defined by:

$$\vec{H} = h_x^i \vec{i} + h_y^i \vec{j} + H_z^i \vec{k}$$

and

$$\vec{M} = m_x \vec{i} + m_y \vec{j} + (M_z + m_z)\vec{k}$$

In solving Equation (1), an exponential, exp $[j\omega t]$, time dependence is assumed for the alternating magnetic field and magnetization, and if the following assumption is made:

$$h_x, h_y, h_z \ll H_a$$

it is easily shown that the alternating components of the magnetization of the medium are given by (neglecting terms of the second order in small quantities):

$$\begin{aligned} m_x &= \frac{[\gamma^2 M_z H_z^i(1 + \alpha^2) + j\gamma\alpha M_z\omega]h_x^i - j\gamma M_z \omega h_y^i}{\gamma^2 H_z^{i2}(1 + \alpha^2) - \omega^2 + j[2\omega\gamma\alpha H_z^i]} \\ m_y &= \frac{[\gamma^2 M_z H_z^i(1 + \alpha^2) + j\gamma\alpha M_z\omega]h_y^i + j\gamma M_z \omega h_x^i}{\gamma^2 H_z^{i2}(1 + \alpha^2) - \omega^2 + j[2\omega\gamma\alpha H_z^i]} \\ m_z &= 0 \end{aligned} \tag{2}$$

where:

$$j = \sqrt{-1}$$

Since

$$\vec{b} = \vec{h}^i + 4\pi\vec{m}, \tag{3}$$

it is possible by means of Equations (2) and (3) to find the relation between the alternating flux density $\vec{b}$ and the internal alternating field $\vec{h}^i$. If the ferromagnetic body is considered as being infinite, the internal fields and applied fields are equal. Hence, for this case:

$$\begin{aligned} b_x &= \mu h_x - jKh_y \\ b_y &= jKh_x + \mu h_y \\ b_z &= h_z \end{aligned} \tag{4}$$

where:

$$\mu = \mu' - j\mu''$$

$$K = K' - jK''$$

and:

$$\mu' = 1 + \frac{[\gamma^2 H_a^2(1 + \alpha^2) - \omega^2][4\pi M_z \gamma^2 H_a(1 + \alpha^2)] + 8\pi M_z \omega^2 \gamma^2 \alpha^2 H_a}{[\gamma^2 H_a^2(1 + \alpha^2) - \omega^2]^2 + 4\omega^2 \gamma^2 \alpha^2 H_a^2}$$

$$K' = \frac{4\pi M_z \gamma \omega [\gamma^2 H_a^2(1 + \alpha^2) - \omega^2]}{[\gamma^2 H_a^2(1 + \alpha^2) - \omega^2]^2 + 4\omega^2 \gamma^2 \alpha^2 H_a^2}$$

$$K'' = \frac{8\pi M_z \omega^2 \gamma^2 \alpha H_a}{[\gamma^2 H_a^2(1 + \alpha^2) - \omega^2]^2 + 4\omega^2 \gamma^3 \alpha^2 H_a^2}$$

$$\mu'' = \frac{4\pi M_z \gamma \alpha \omega [\gamma^2 H_a^2(1 + \alpha^2) + \omega^2]}{[\gamma^2 H_a^2(1 + \alpha^2) - \omega^2]^2 + 4\omega^2 \gamma^2 \alpha^2 H_a^2}$$

In order to find the behavior of a wave being propagated in this medium, it is necessary to find a solution to Maxwell's equations which are consistent with the above set of equations and in which, b, h, E, and D are of the following form:

$$\begin{aligned} \vec{b} &= \vec{b}_0 \exp [j\omega t - \Gamma(\vec{n} \cdot \vec{r})] \\ \vec{h} &= \vec{h}_0 \exp [j\omega t - \Gamma(\vec{n} \cdot \vec{r})] \\ \vec{E} &= \vec{E}_0 \exp [j\omega t - \Gamma(\vec{n} \cdot \vec{r})] \\ \vec{D} &= \vec{D}_0 \exp [j\omega t - \Gamma(\vec{n} \cdot \vec{r})] \end{aligned} \tag{5}$$

where $\vec{E}_0$, and $\vec{h}_0$ are complex vector functions of the coordinates and which satisfy the boundary conditions imposed by the waveguide. Further:

$\vec{n}$ = unit vector in the direction of propagation

Γ = propagation constant

Maxwell's equations are:

$$\begin{aligned} \nabla \times \vec{E} &= -\frac{1}{c}\frac{\partial \vec{b}}{\partial t} \\ \nabla \times \vec{h} &= \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \end{aligned} \tag{6}$$

Inserting the values given in Equations (5), these become:

$$\nabla \times \vec{E}_0 - \Gamma(\vec{n} \times \vec{E}_0) = \frac{-j\omega \vec{b}_0}{c} \tag{7}$$

$$\nabla \times \vec{h}_0 - \Gamma(\vec{n} \times \vec{h}_0) = \frac{j\omega\epsilon \vec{E}_0}{c} \tag{8}$$

which can be combined to:

$$\frac{\omega^2\varepsilon}{c^2}\vec{b}_0 = \nabla \times (\nabla \times \vec{h}_0 - \Gamma\vec{n} \times \vec{h}_0) - \Gamma\vec{n} \times (\nabla \times \vec{h}_0 - \Gamma\vec{n} \times \vec{h}_0) \tag{9}$$

Writing Equation (9) in component form gives:

$$\begin{aligned}\frac{\omega^2\varepsilon}{c^2} b_x = {} & \frac{\partial^2 h_y}{\partial y \partial x} + \frac{\partial^2 h_z}{\partial z \partial x} - \frac{\partial^2 h_x}{\partial y^2} - \frac{\partial^2 h_x}{\partial z^2} - \Gamma n_x \left(\frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z}\right) \\ & + 2\Gamma\left(n_y \frac{\partial h_x}{\partial y} + n_z \frac{\partial h_x}{\partial z}\right) - \Gamma\left(n_y \frac{\partial h_y}{\partial x} + n_z \frac{\partial h_z}{\partial x}\right) \\ & + \Gamma^2 n_x(n_x h_x + n_y h_y + n_z h_z) - \Gamma^2 h_x\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{\omega^2\varepsilon}{c^2} b_y = {} & \frac{\partial^2 h_x}{\partial y \partial x} + \frac{\partial^2 h_z}{\partial y \partial z} - \frac{\partial^2 h_y}{\partial x^2} - \frac{\partial^2 h_y}{\partial z^2} - \Gamma n_y \left(\frac{\partial h_x}{\partial x} + \frac{\partial h_z}{\partial z}\right) \\ & + 2\Gamma\left(n_x \frac{\partial h_y}{\partial x} + n_z \frac{\partial h_y}{\partial z}\right) - \Gamma\left(n_x \frac{\partial h_x}{\partial y} + n_z \frac{\partial h_z}{\partial y}\right) \\ & + \Gamma^2 n_y(n_x h_x + n_y h_y + n_z h_z) - \Gamma^2 h_y\end{aligned} \tag{11}$$

$$\begin{aligned}\frac{\omega^2\varepsilon}{c^2} b_z = {} & \frac{\partial^2 h_x}{\partial z \partial x} + \frac{\partial^2 h_y}{\partial z \partial y} - \frac{\partial^2 h_z}{\partial x^2} - \frac{\partial^2 h_z}{\partial y^2} \\ & - \Gamma n_z \left(\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y}\right) + 2\Gamma\left(n_x \frac{\partial h_z}{\partial x} + n_y \frac{\partial h_z}{\partial y}\right) \\ & - \Gamma\left(n_x \frac{\partial h_x}{\partial z} + n_y \frac{\partial h_y}{\partial z}\right) + \Gamma^2 n_z(n_x h_x + n_y h_y + n_z h_z) \\ & - \Gamma^2 h_z\end{aligned} \tag{12}$$

where the subscript 0 has been dropped from all components for convenience.

If the wave in question is an infinite plane wave being propagated along the z axis, then:

$$n_x = n_y = 0, \qquad n_z = 1$$

and the components of $\vec{h}_0$ are constants, and h_z is zero. For this particular case, Equations (10), (11) and (12) become:

$$\frac{\omega^2\varepsilon}{c^2} b_x = -\Gamma^2 h_x \tag{13}$$

$$\frac{\omega^2\varepsilon}{c^2} b_y = -\Gamma^2 h_y \tag{14}$$

Equations (13) and (14) are general differential equations derivable from Maxwell's equations and do not yet contain the properties of any particular medium. In order to find the behavior of a wave travelling through an infinite ferromagnetic medium which is magnetized along the direction of propagation, it is necessary to combine these equations with Equations (4) which describe the relation between b and h in the medium.

This gives:

$$(\mu h_x - jKh_y)\frac{\omega^2 \varepsilon}{c^2} = -\Gamma^2 h_x \tag{15}$$

$$(\mu h_y + jKh_x)\frac{\omega^2 \varepsilon}{c^2} = -\Gamma^2 h_y \tag{16}$$

The only possible solution to this set of equations is a circularly polarized wave where:

$$h_x = \pm j h_y$$

The positive sign above represents a so-called positive circularly polarized wave and the negative sign a negative circularly polarized wave. The propagation constants for these waves is given by

$$\Gamma_{\pm} = \frac{j\omega}{c}\sqrt{\varepsilon(\mu \pm K)} \tag{17}$$

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